



Examples of Supervisory Interaction with Route Optimizers

Oliver Turnbull and Arthur Richards

oliver.turnbull@bristol.ac.uk

arthur.richards@bristol.ac.uk

Overview

- The SESAR Concept of Operations calls for:
“Extensive use of automation support to reduce operator task load, but in which controllers remain in control as managers”
- Much work has been performed on trajectory optimization
- Rarely are humans included in the trajectory design process
- Part of SUPEROPT project whose goal is to:
“Develop tools to facilitate interactions between humans and trajectory optimizers”
- Trajectory optimization can play a key role in automation support

Overview

- **How** do we facilitate supervisor interaction?

Overview

- **How** do we facilitate supervisor interaction?

Sense Constraints

- **What** are sense constraints?
- **Why** are they useful?

Outline

1. MILP

- Fast
- Global optimum
- Linearized
- 3D dynamics model
- Extension of sense constraints to 3D

2. Collocation with Polar Sets

- Nonlinear model
- More general problems, eg noise as cost
- Explicit modelling of time
- 4D obstacles

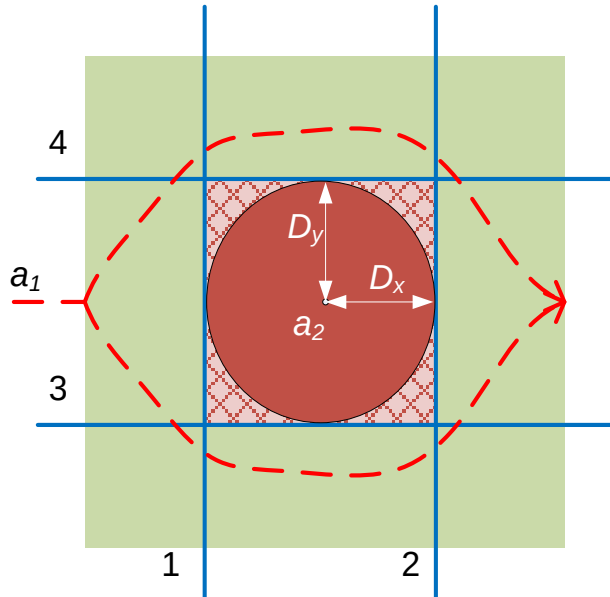
3. Conclusions

Assumptions

- 4-D trajectories (RBTs)
- Trajectories updated via data-link
- Free routing



MILP Obstacle Avoidance



- Approximate obstacle with multiple avoidance constraints

$$r_x(a_2, k_1) - r_x(a_1, k_1) \geq D_x$$

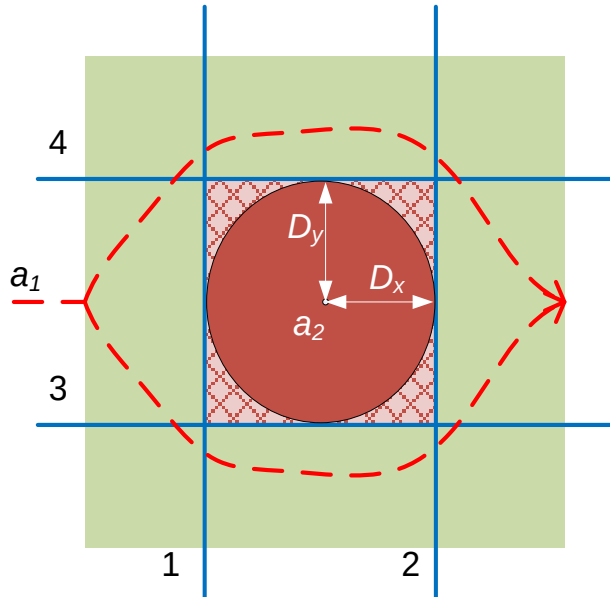
$$r_x(a_2, k_1) - r_x(a_1, k_1) \geq -D_x$$

$$r_y(a_2, k_1) - r_y(a_1, k_1) \geq D_y$$

$$r_y(a_2, k_1) - r_y(a_1, k_1) \geq -D_y$$

$$\forall k_1 \in \{1, \dots, N_t\}, k_2 \in \{1, \dots, N_t\} : (k_1 \geq k_2)$$

MILP Obstacle Avoidance



- Approximate obstacle with multiple avoidance constraints
- Define “binary” variables that enable each avoidance constraint to be relaxed

$$r_x(a_2, k_1) - r_x(a_1, k_1) \geq D_x + Mb_a(a_1, a_2, k_1, k_2, 1)$$

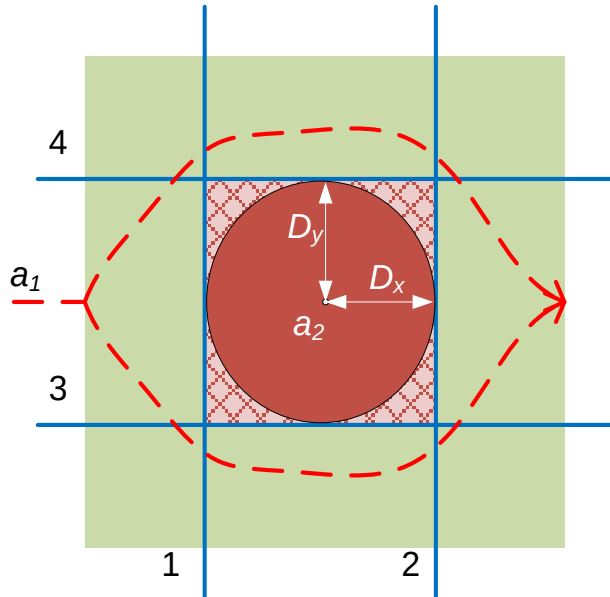
$$r_x(a_2, k_1) - r_x(a_1, k_1) \geq -D_x - Mb_a(a_1, a_2, k_1, k_2, 2)$$

$$r_y(a_2, k_1) - r_y(a_1, k_1) \geq D_y + Mb_a(a_1, a_2, k_1, k_2, 3)$$

$$r_y(a_2, k_1) - r_y(a_1, k_1) \geq -D_y - Mb_a(a_1, a_2, k_1, k_2, 4)$$

$$\forall k_1 \in \{1, \dots, N_t\}, k_2 \in \{1, \dots, N_t\} : (k_1 \geq k_2)$$

MILP Obstacle Avoidance



- Approximate obstacle with multiple avoidance constraints
- Define “binary” variables that enable each avoidance constraint to be relaxed
- Require at least one of the constraints to be enforced

$$r_x(a_2, k_1) - r_x(a_1, k_1) \geq D_x + Mb_a(a_1, a_2, k_1, k_2, 1)$$

$$r_x(a_2, k_1) - r_x(a_1, k_1) \geq -D_x - Mb_a(a_1, a_2, k_1, k_2, 2)$$

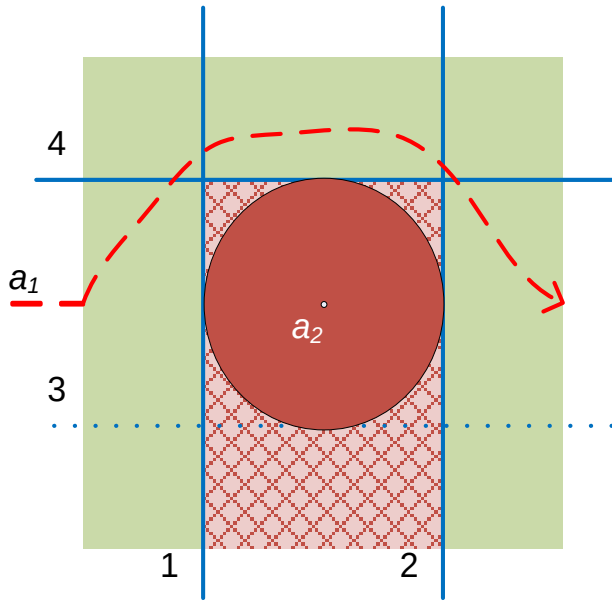
$$r_y(a_2, k_1) - r_y(a_1, k_1) \geq D_y + Mb_a(a_1, a_2, k_1, k_2, 3)$$

$$r_y(a_2, k_1) - r_y(a_1, k_1) \geq -D_y - Mb_a(a_1, a_2, k_1, k_2, 4)$$

$$\sum_{i=1}^4 b_a(a_1, a_2, k_1, k_2, i) \leq 3$$

$$\forall k_1 \in \{1, \dots, N_t\}, k_2 \in \{1, \dots, N_t\} : (k_1 \geq k_2)$$

MILP Sense Constraints



- We can force a trajectory to pass to one side of an obstacle by “freezing” the appropriate binary:

$$r_x(a_2, k_1) - r_x(a_1, k_1) \geq D_x + Mb_a(a_1, a_2, k_1, k_2, 1)$$

$$r_x(a_2, k_1) - r_x(a_1, k_1) \geq -D_x - Mb_a(a_1, a_2, k_1, k_2, 2)$$

$$r_y(a_2, k_1) - r_y(a_1, k_1) \geq D_y + Mb_a(a_1, a_2, k_1, k_2, 3)$$

$$r_y(a_2, k_1) - r_y(a_1, k_1) \geq -D_y - Mb_a(a_1, a_2, k_1, k_2, 4)$$

$$\sum_{i=1}^4 b_a(a_1, a_2, k_1, k_2, i) \leq 3$$

$$b_a(a_1, a_2, k_1, k_2, 3) = 1$$

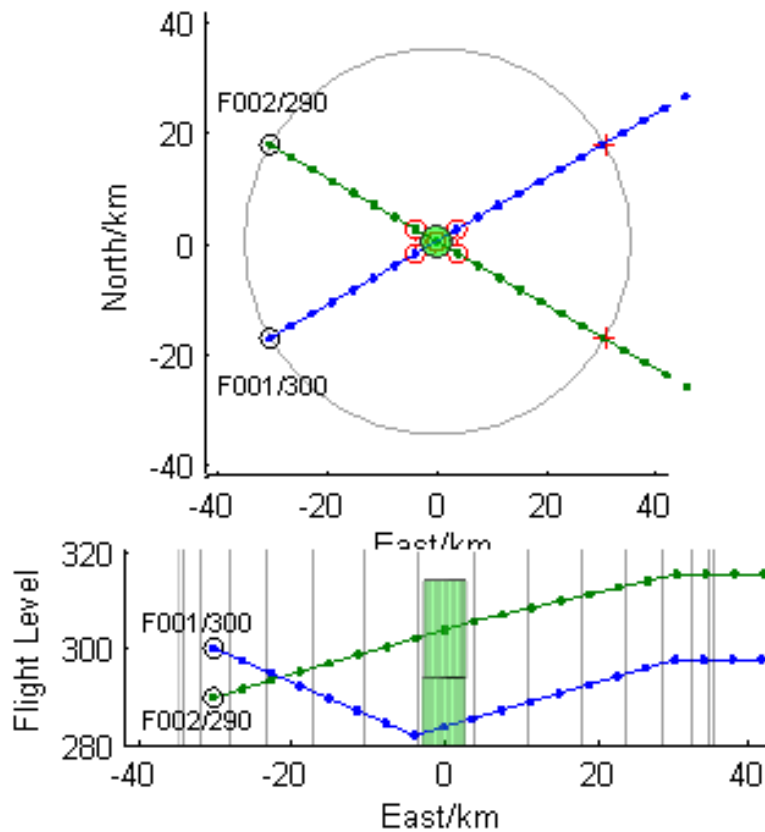
$$\forall k_1 \in \{1, \dots, N_t\}, k_2 \in \{1, \dots, N_t\} : (k_1 \geq k_2)$$

Sense Constraints in ATC

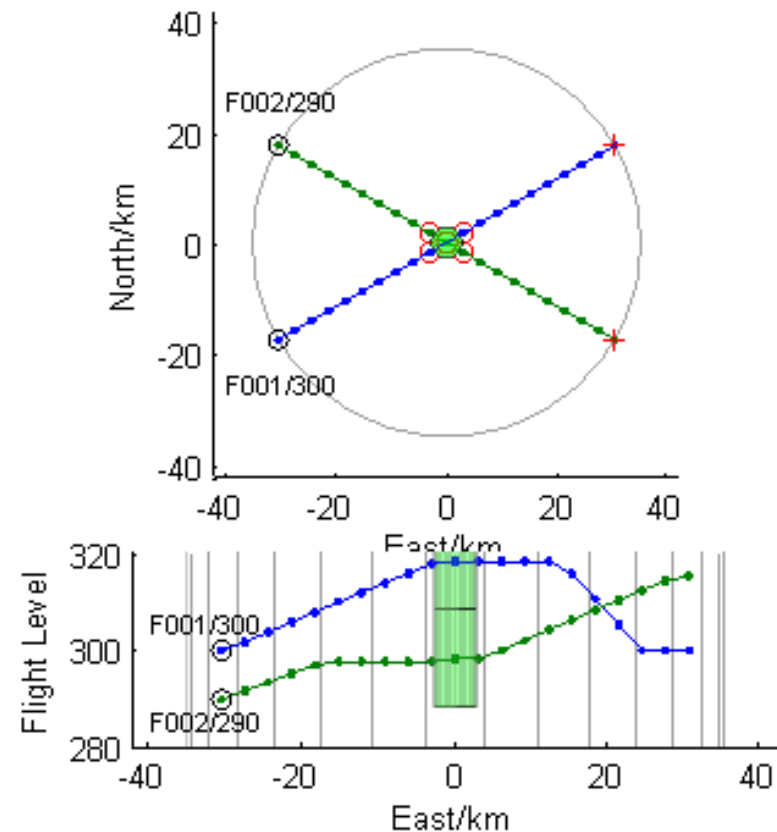
Three Requirements:

1. **Binaries** to define relative position
2. **3-D dynamics model**: derived from BADA
3. **Resolve class of problem**: Fix in 1 or more dimensions.
 - The problem is to resolve **only** in a certain way
 - Other dimensions should not change

Sense Constraints in ATC



F002 over F001
(Unconstrained)



F001 over F002

Sense Constraints in ATC

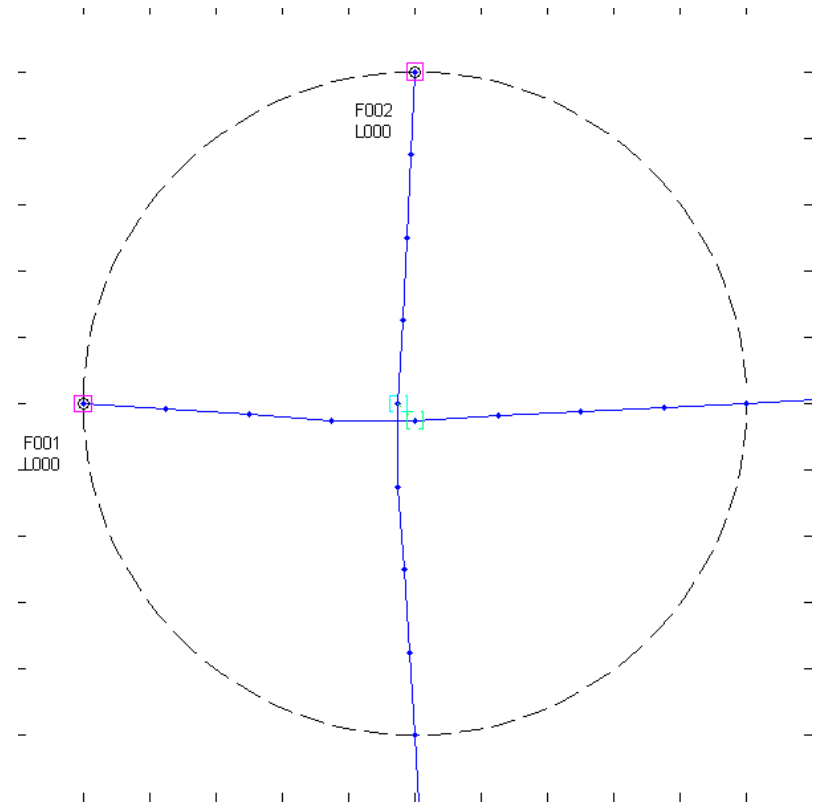
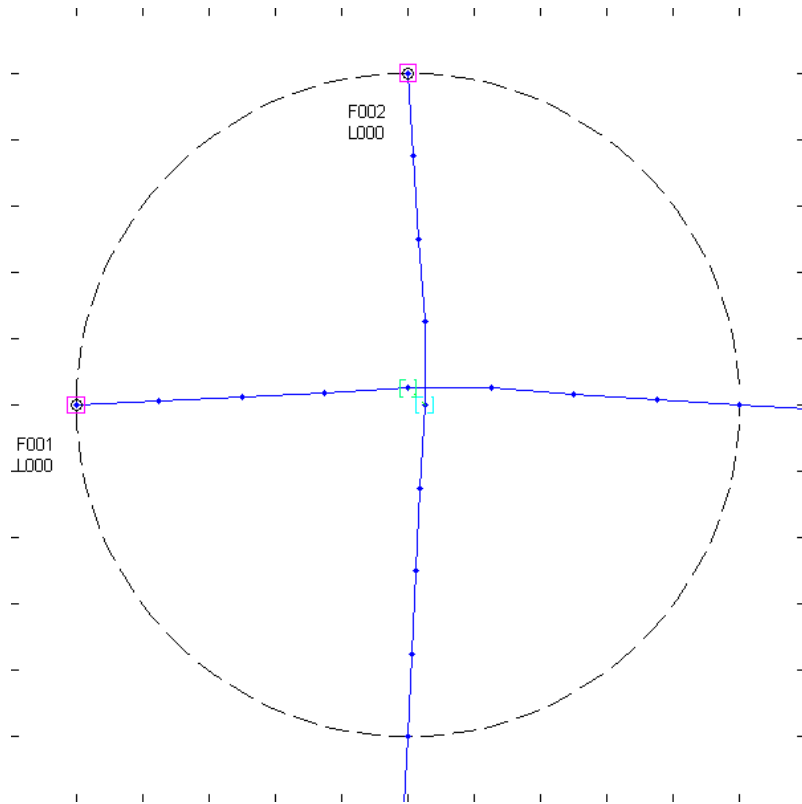
- Vertical:
 - Common notion of **up/down**
- Horizontal
 - Direction (left/right) relative to heading
 - Define as **ahead/behind**
 - Enforced by applying the constraints at the present time and all future time-steps

Sense Constraints in ATC

F002 ahead of F001

F002 behind F001

(Unconstrained; 2D)

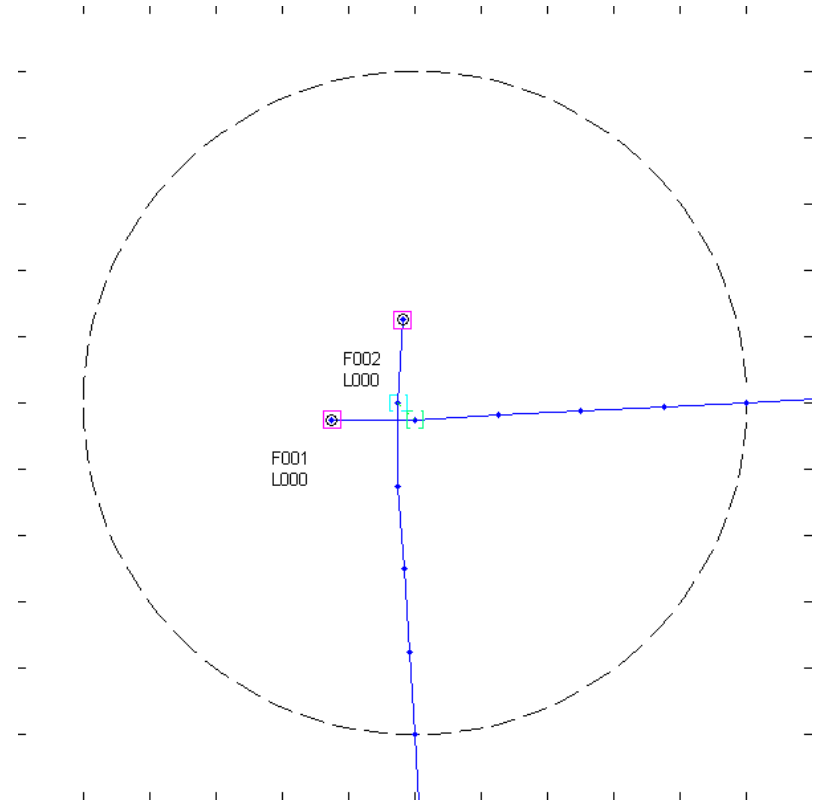
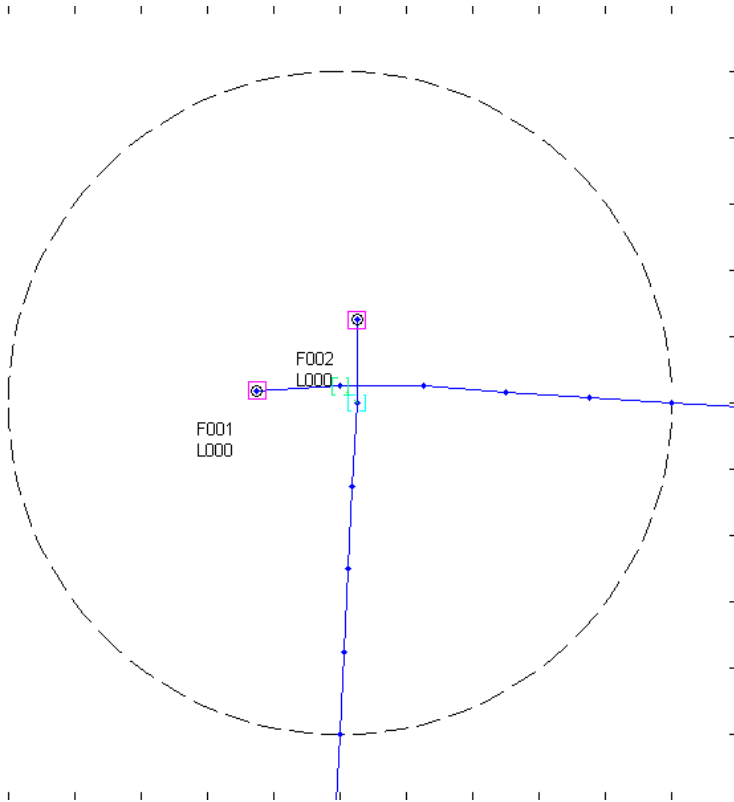


Sense Constraints in ATC

F002 ahead of F001

F002 behind F001

(Unconstrained)

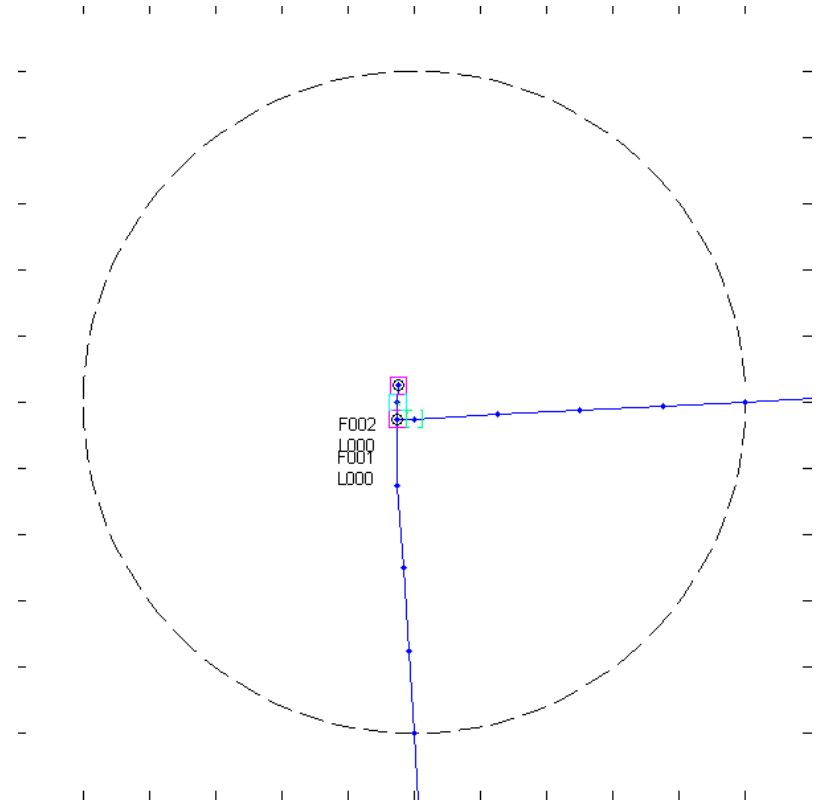
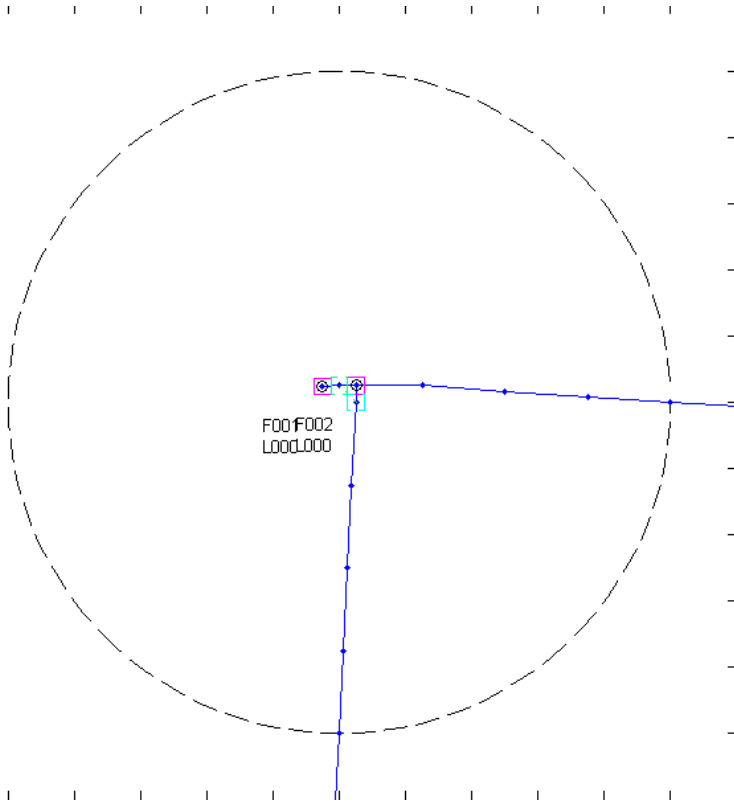


Sense Constraints in ATC

F002 ahead of F001

F002 behind F001

(Unconstrained)

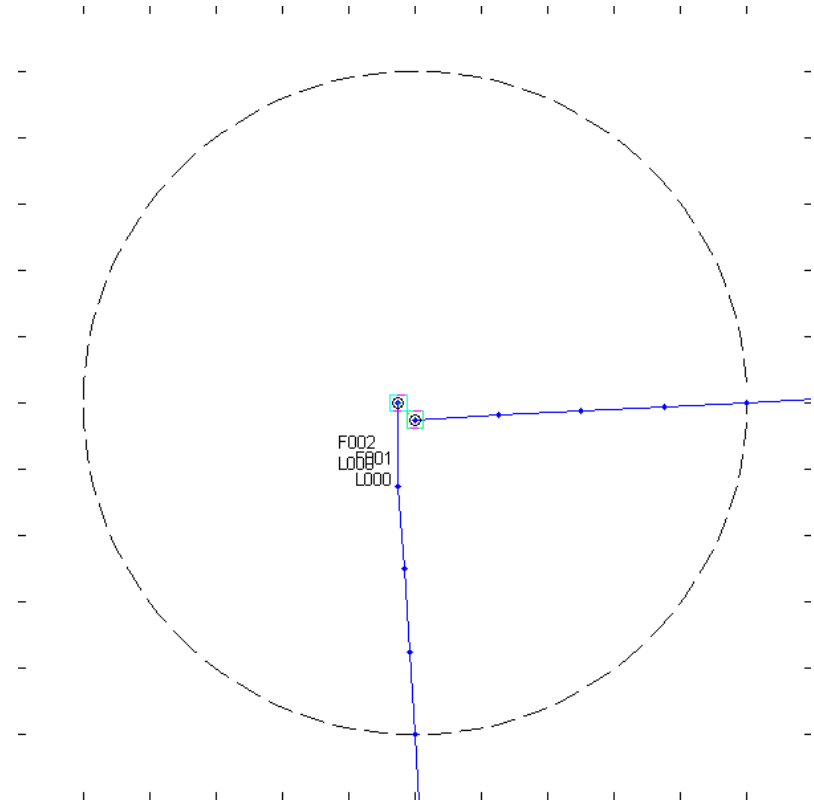
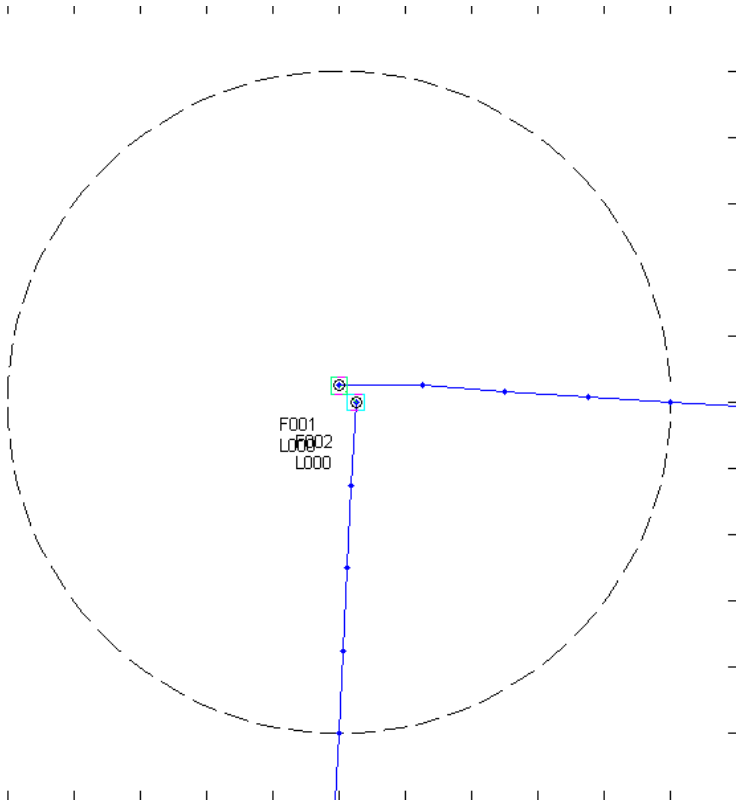


Sense Constraints in ATC

F002 ahead of F001

F002 behind F001

(Unconstrained)

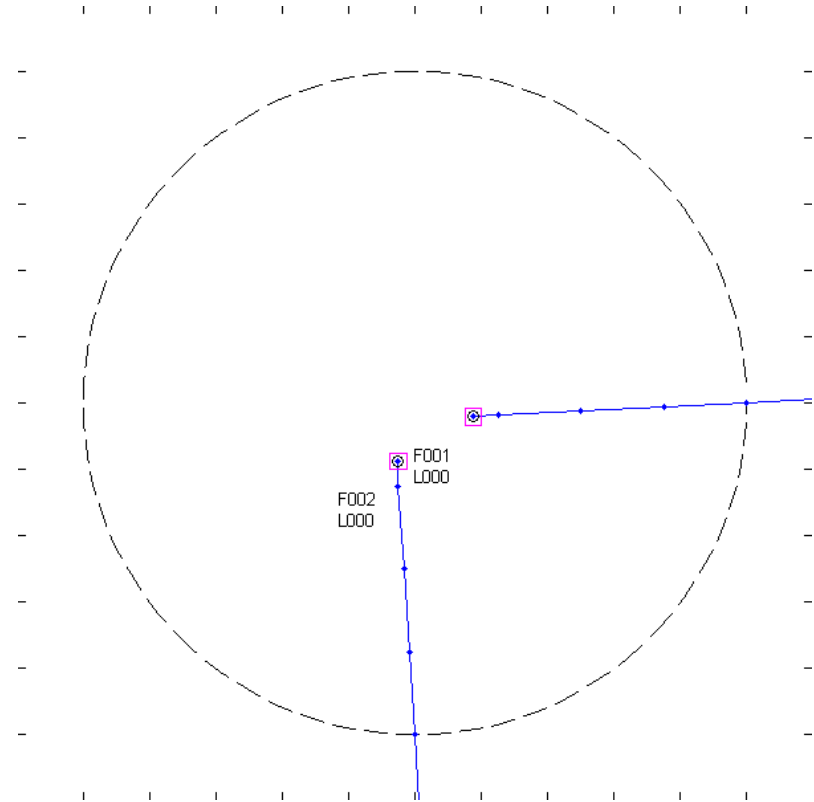
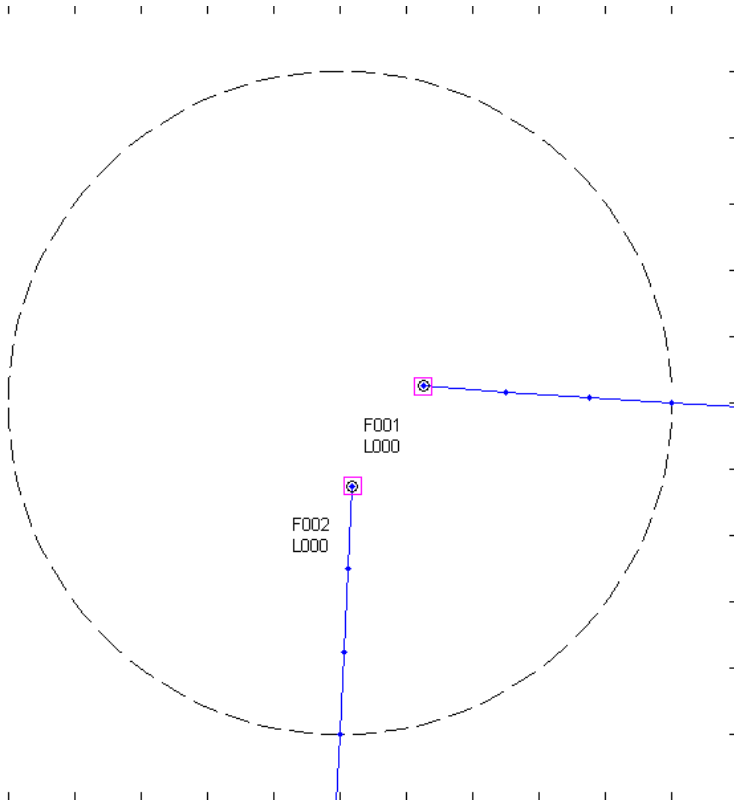


Sense Constraints in ATC

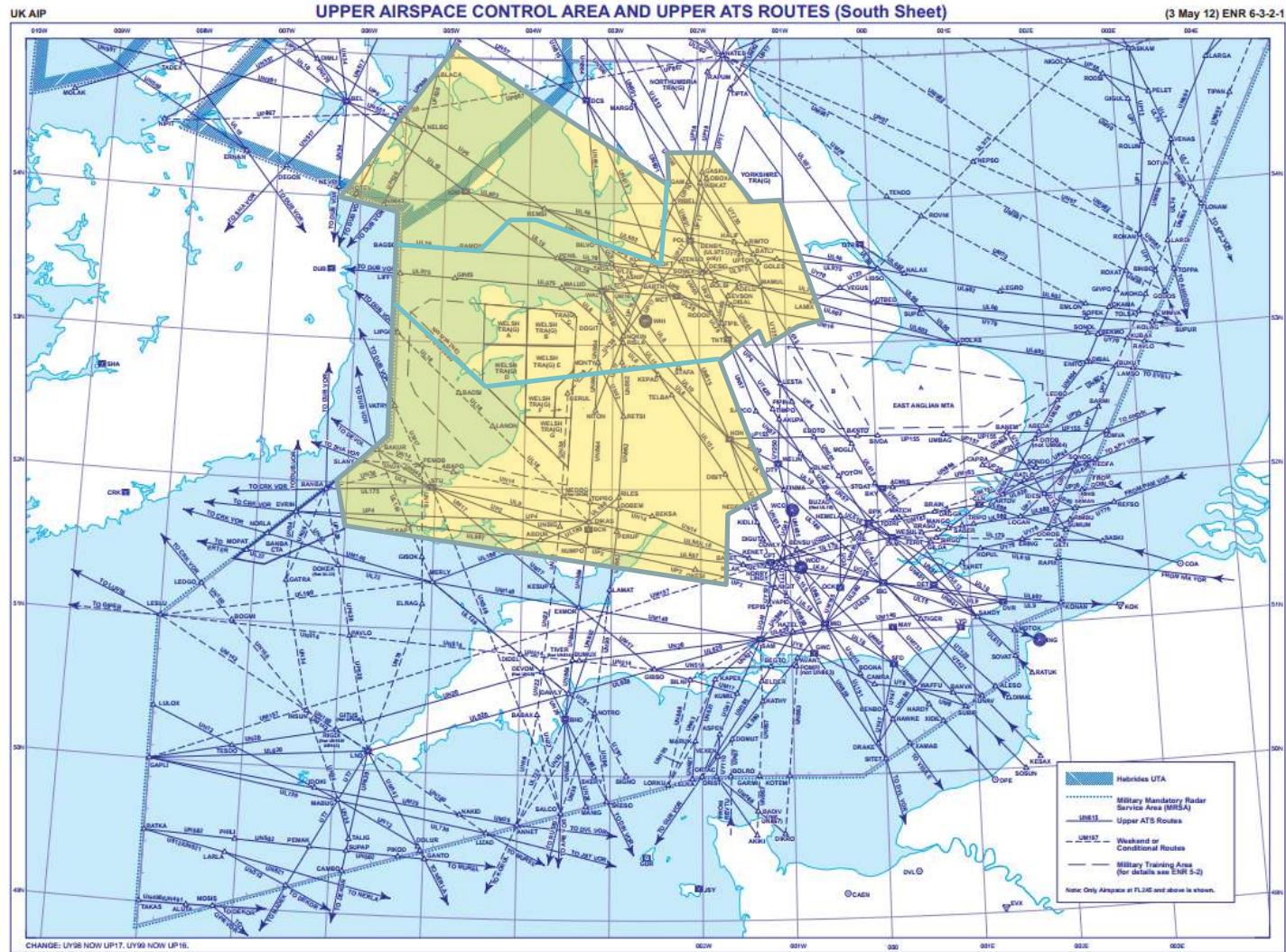
F002 ahead of F001

F002 behind F001

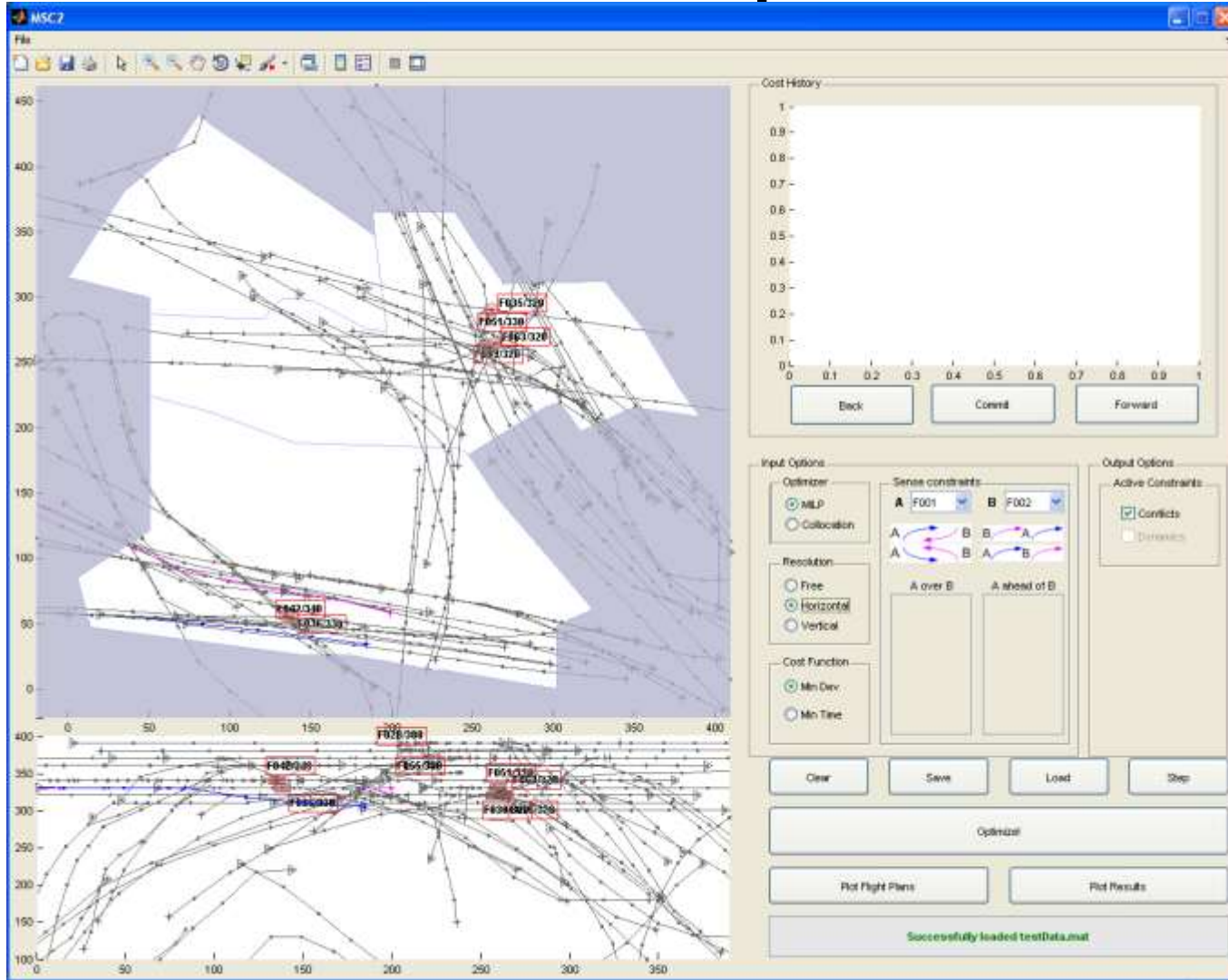
(Unconstrained)



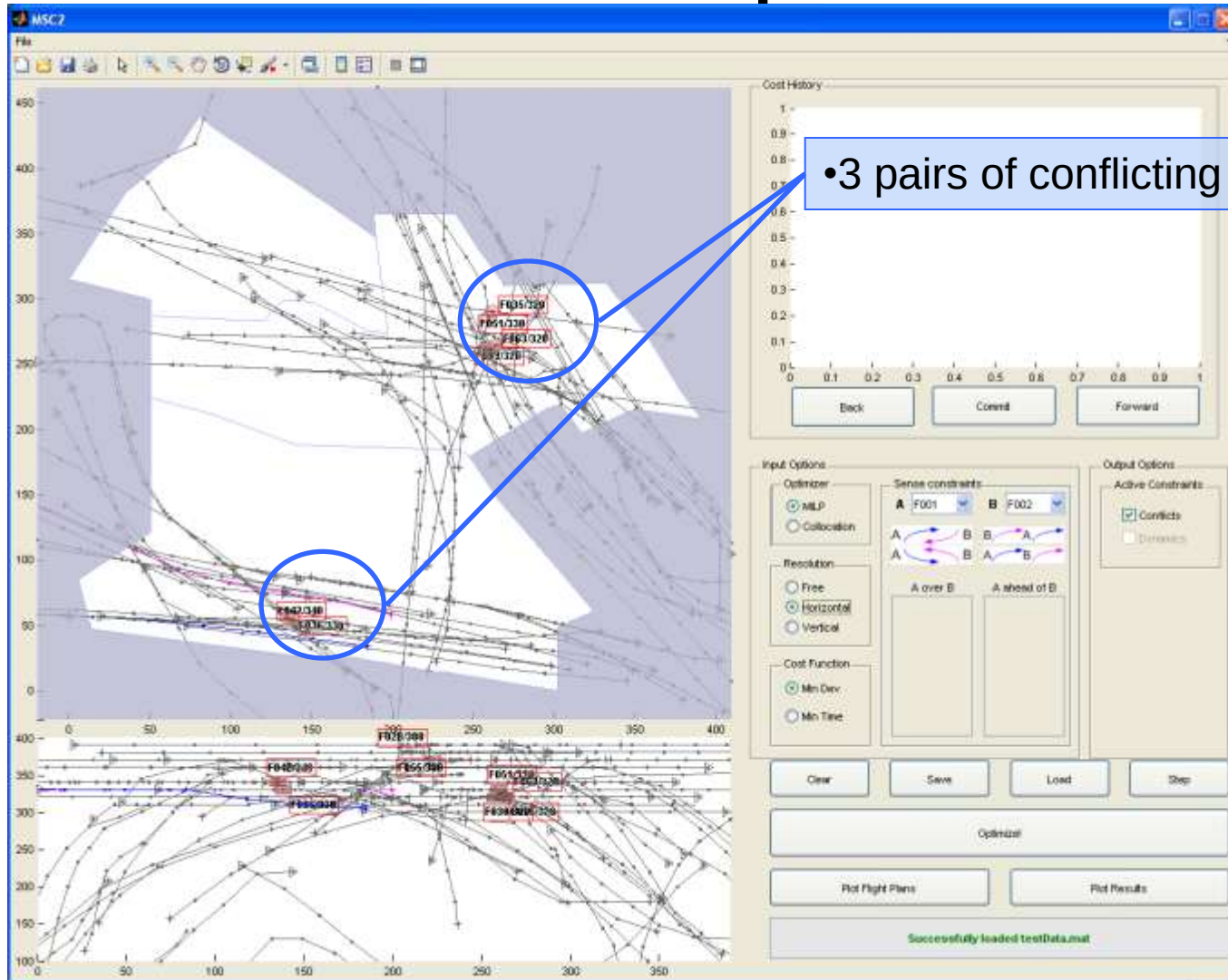
Multi-Sector Controller (MSC)



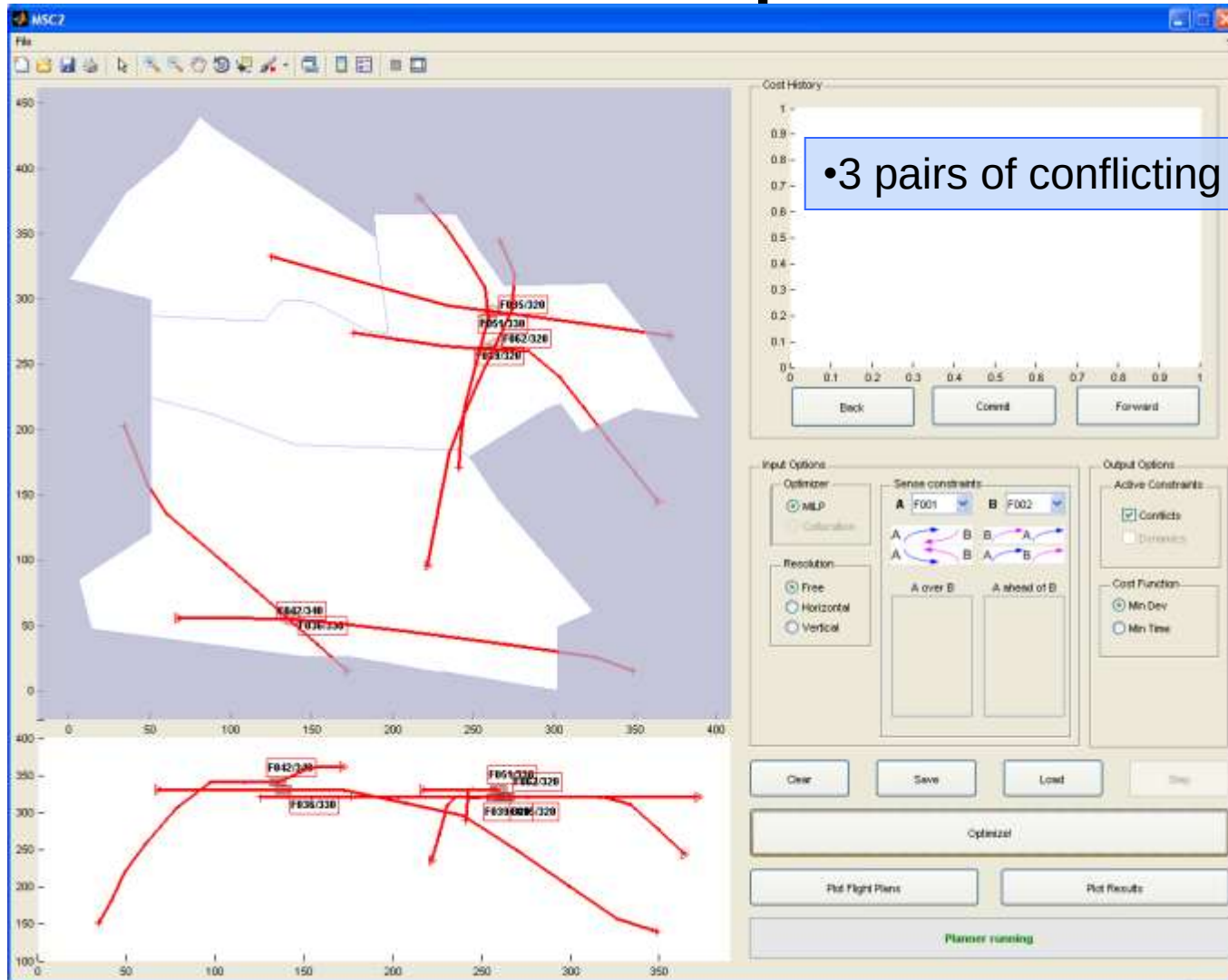
MSC - Input



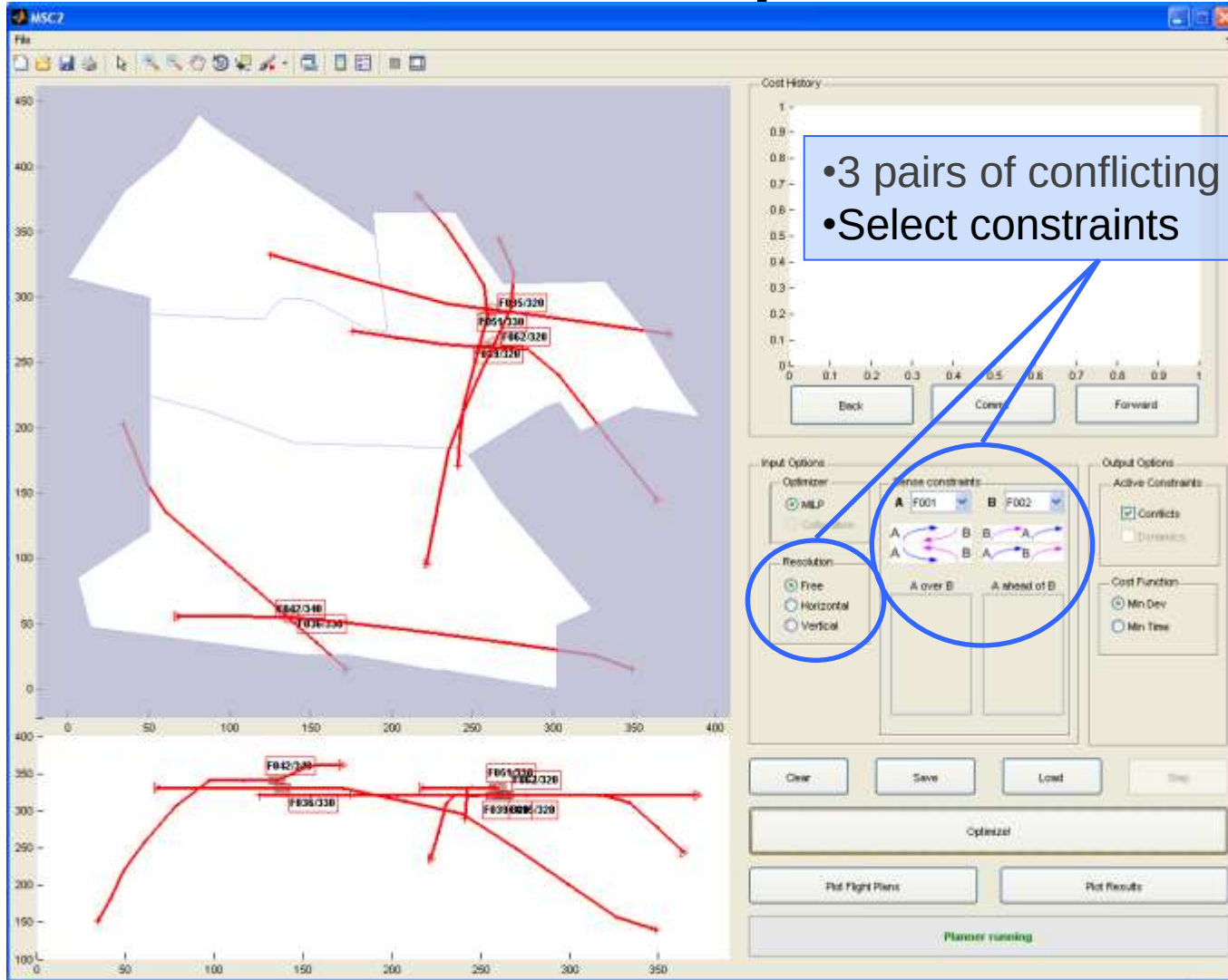
MSC - Input



MSC - Input

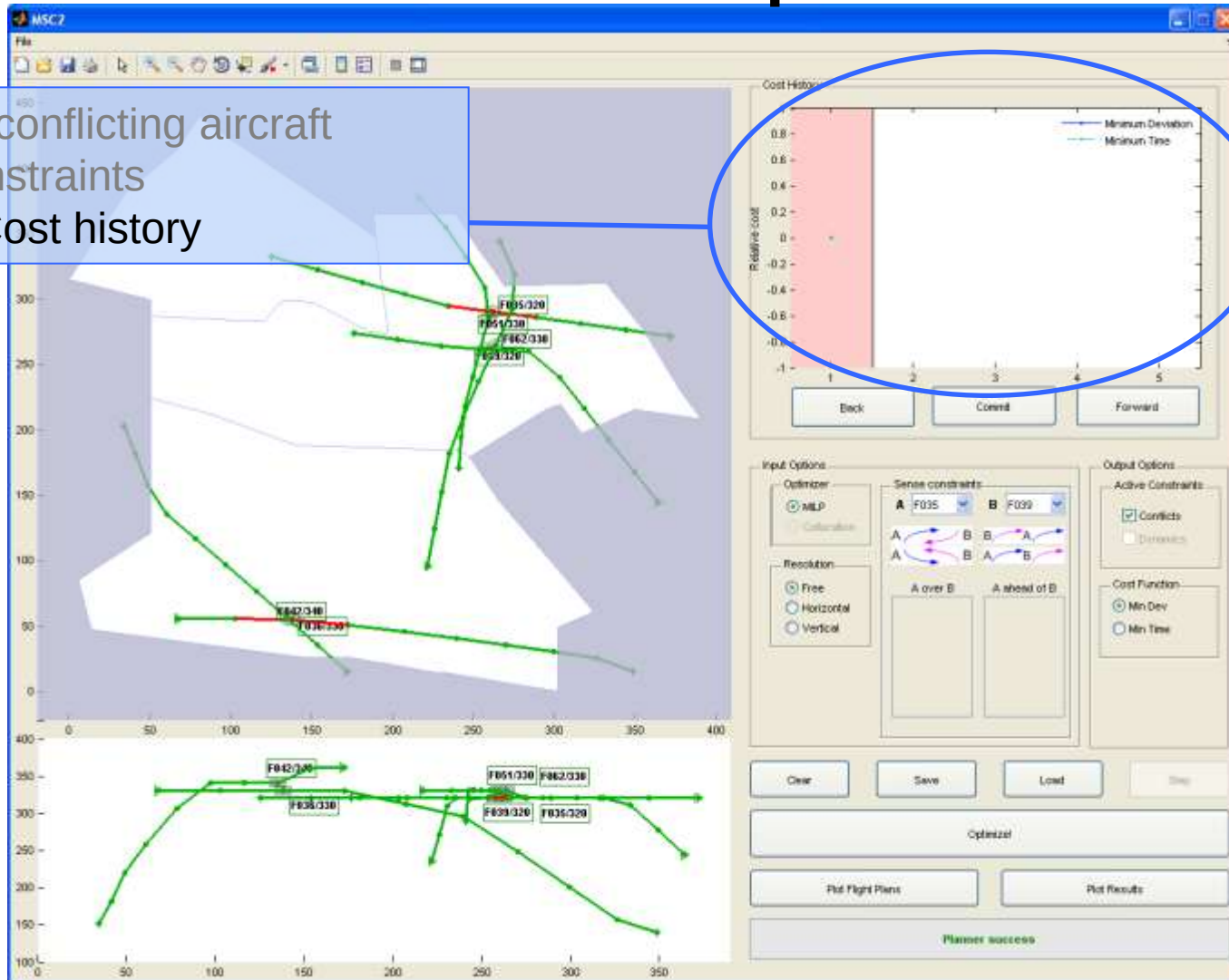


MSC - Input



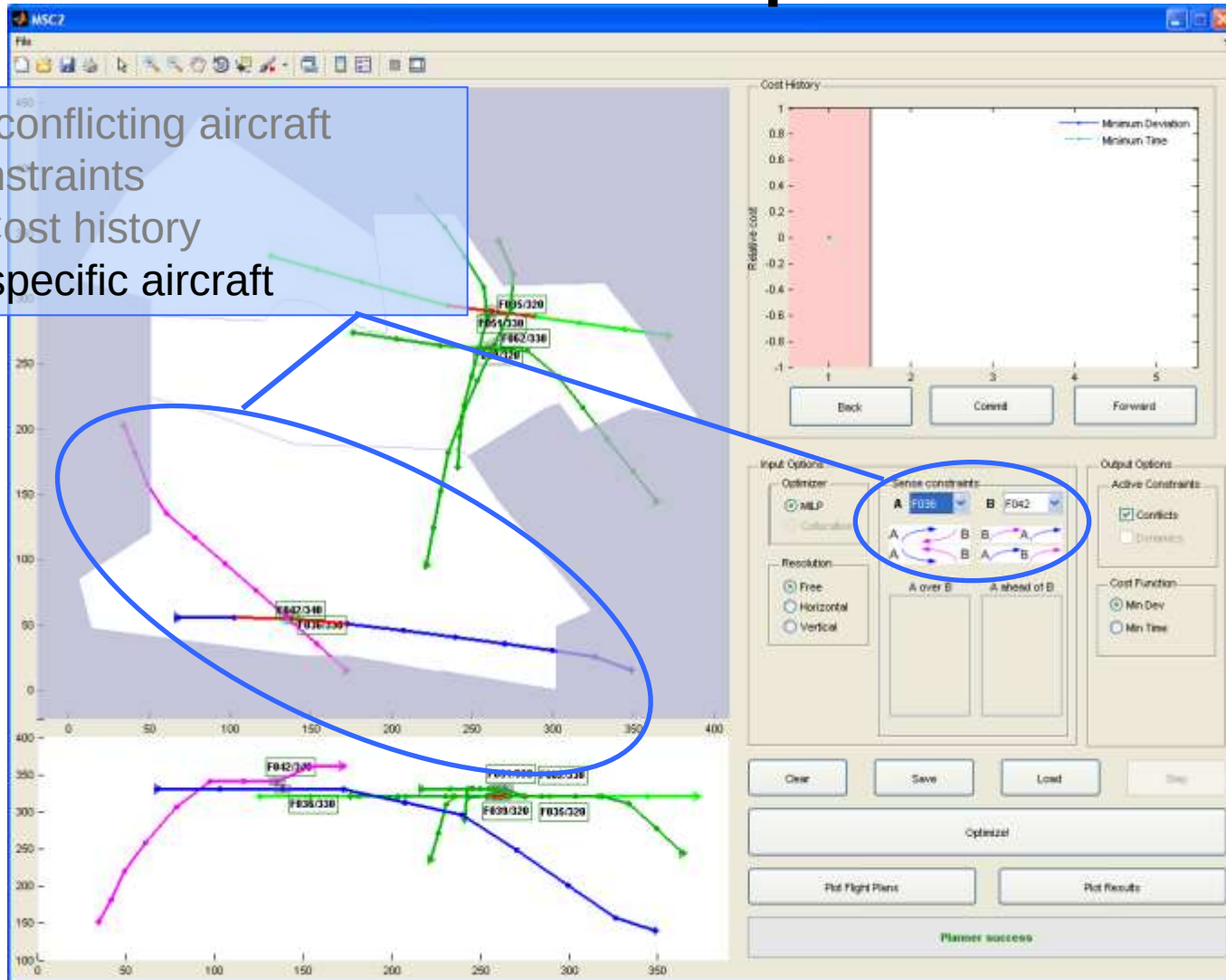
MSC – Input

- 3 pairs of conflicting aircraft
- Select constraints
- Relative Cost history



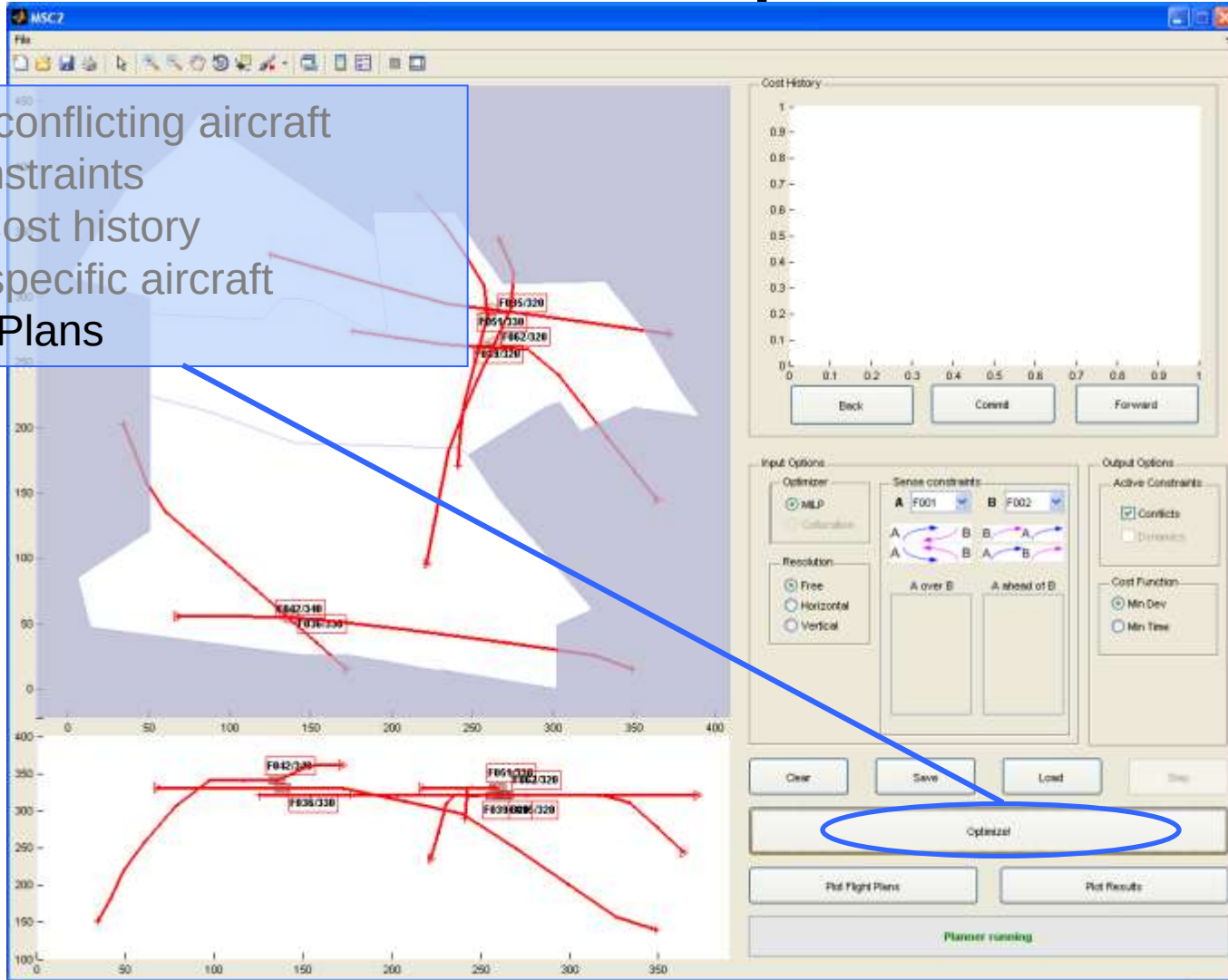
MSC – Input

- 3 pairs of conflicting aircraft
- Select constraints
- Relative Cost history
- Highlight specific aircraft



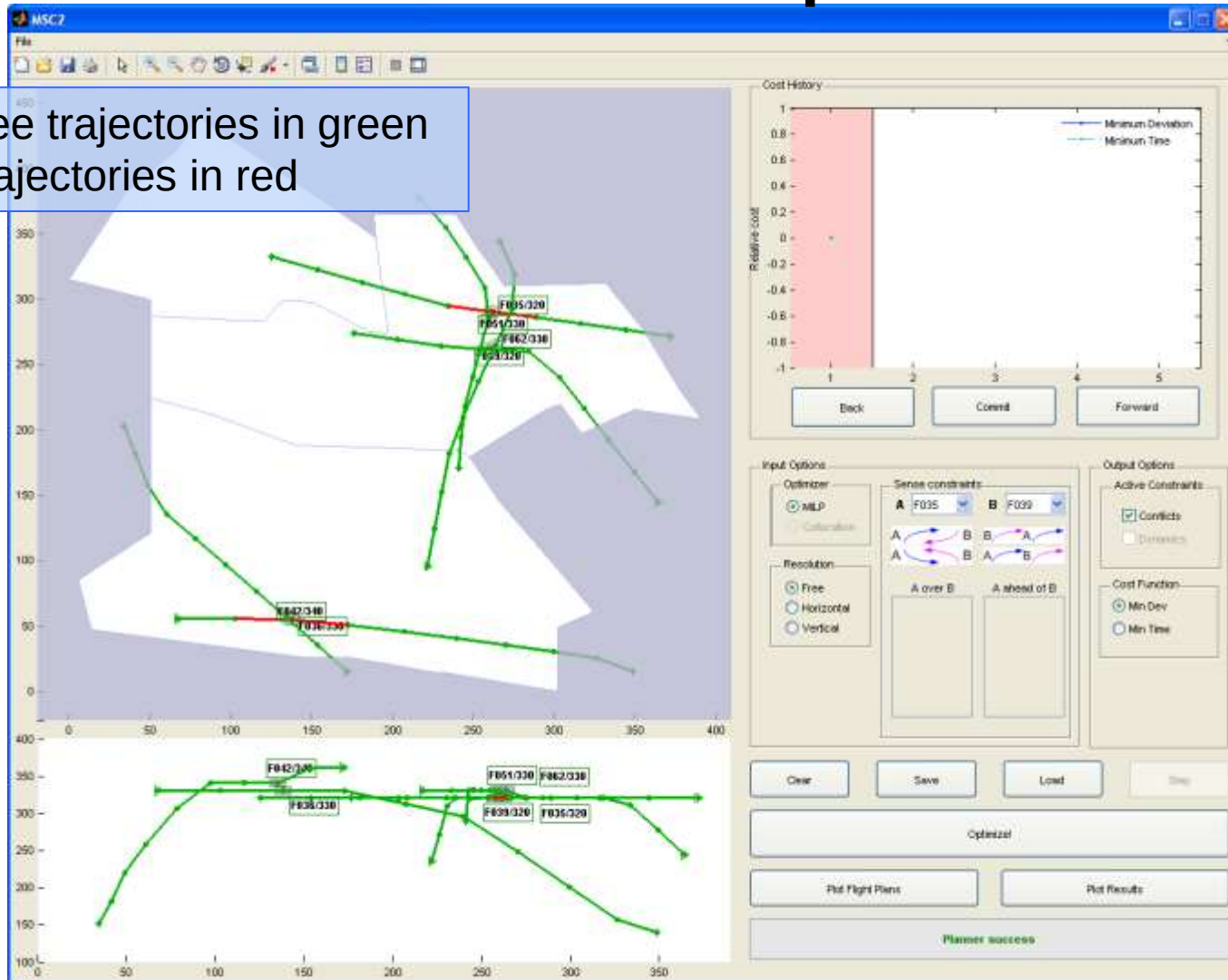
MSC - Input

- 3 pairs of conflicting aircraft
- Select constraints
- Relative Cost history
- Highlight specific aircraft
- Generate Plans

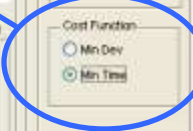


MSC – Step 1

- Conflict free trajectories in green
- Original trajectories in red

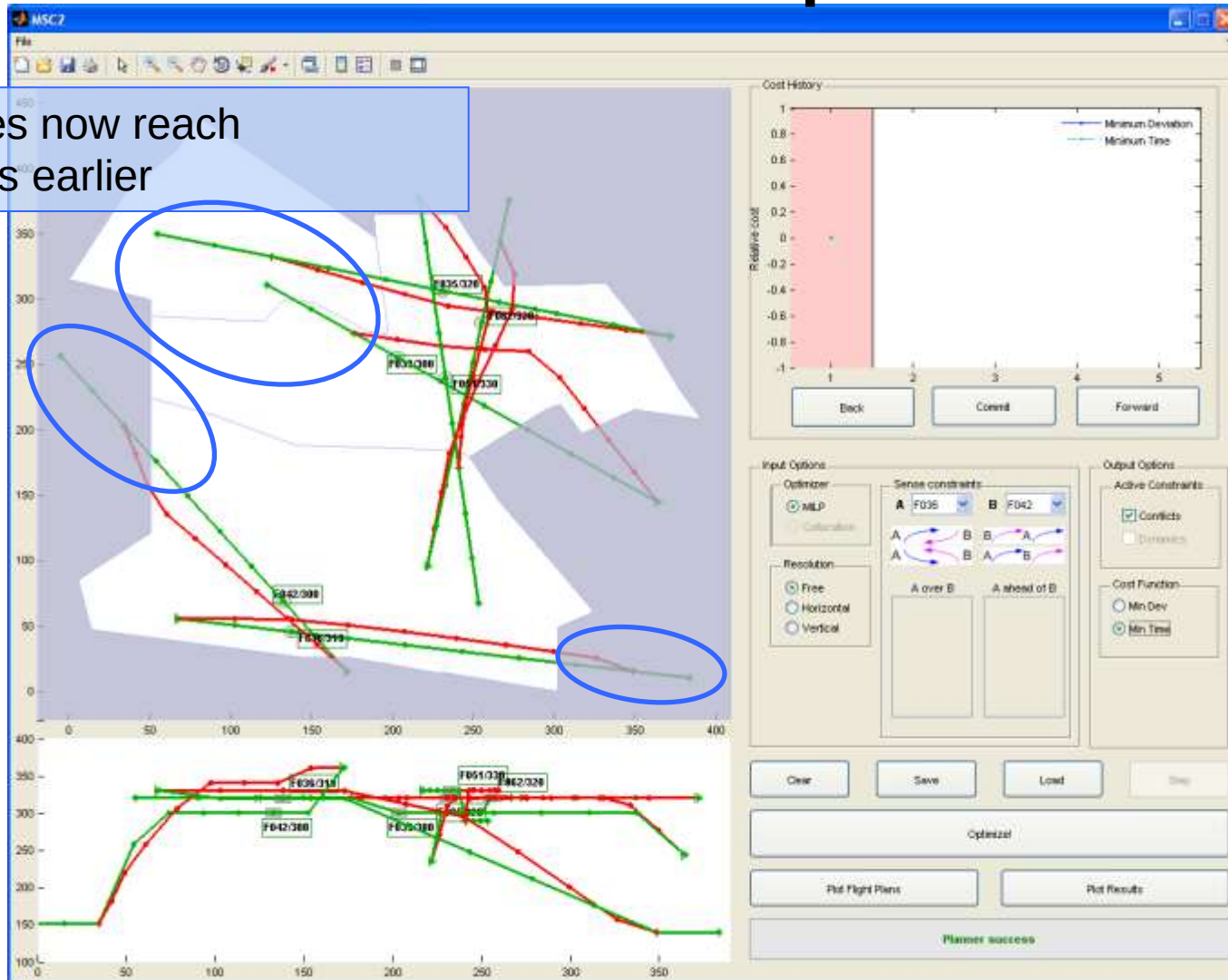


- Highlight specific aircraft
- Alternative cost functions



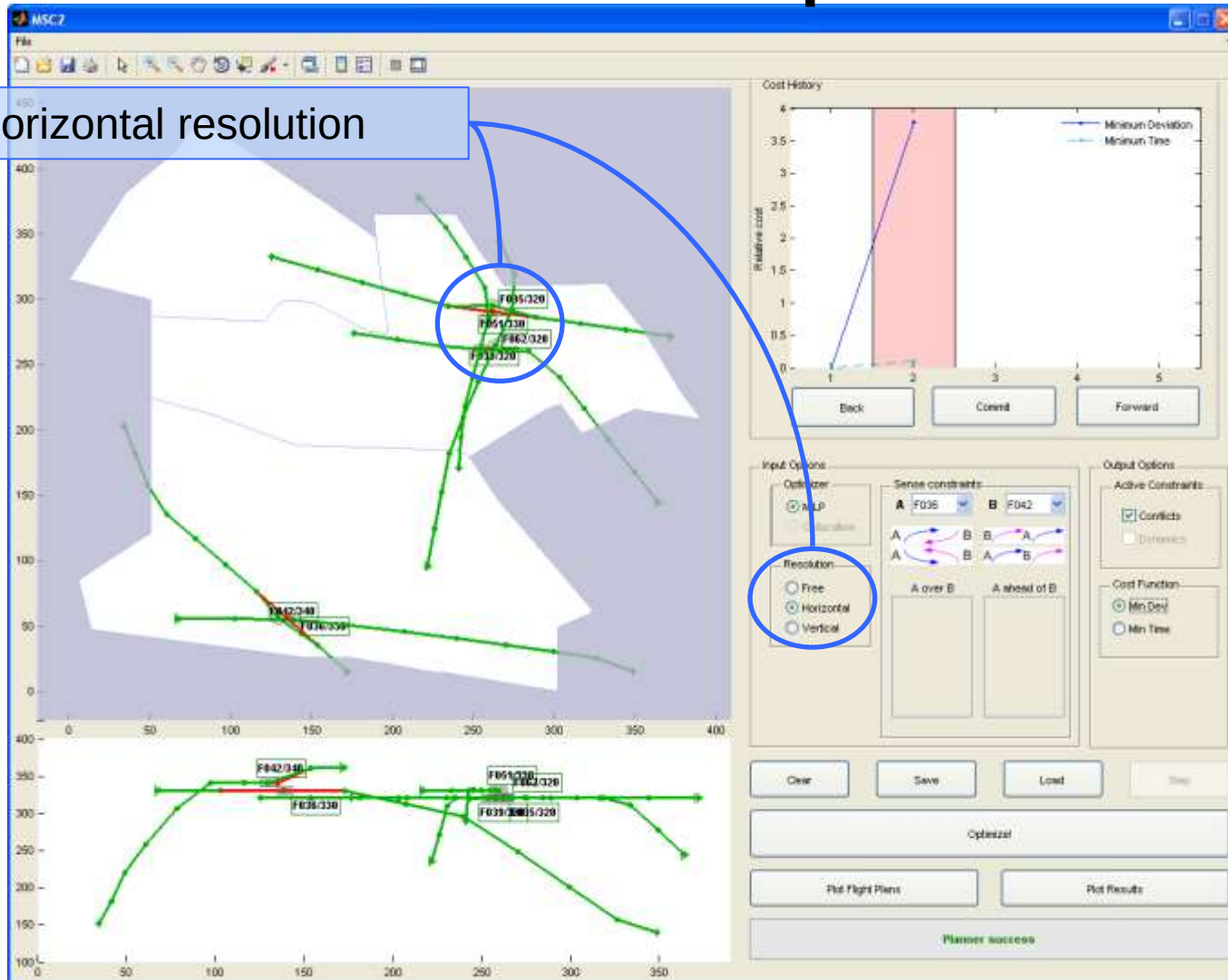
MSC – Step 1

- Trajectories now reach destinations earlier



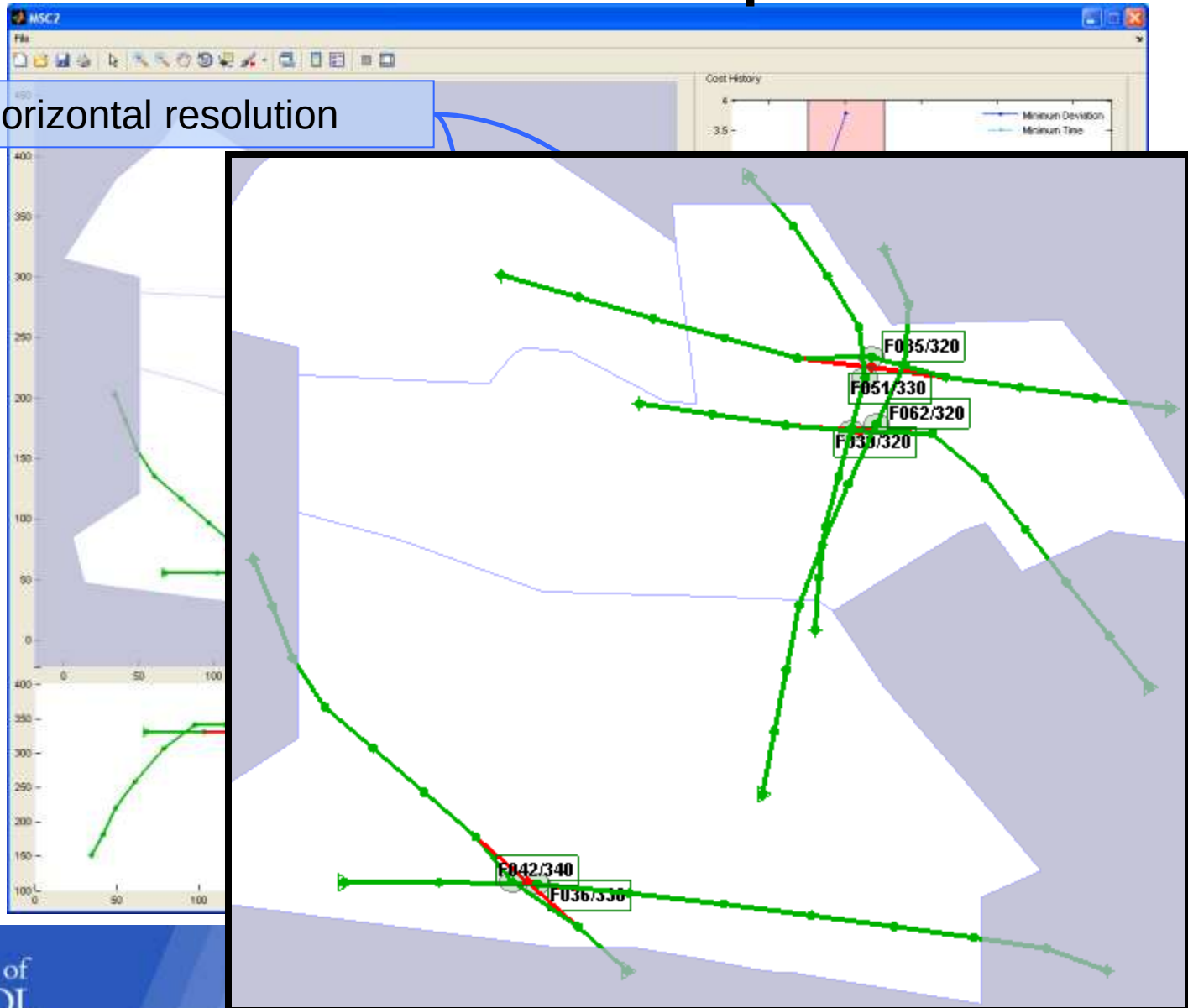
MSC – Step 2

- Request horizontal resolution



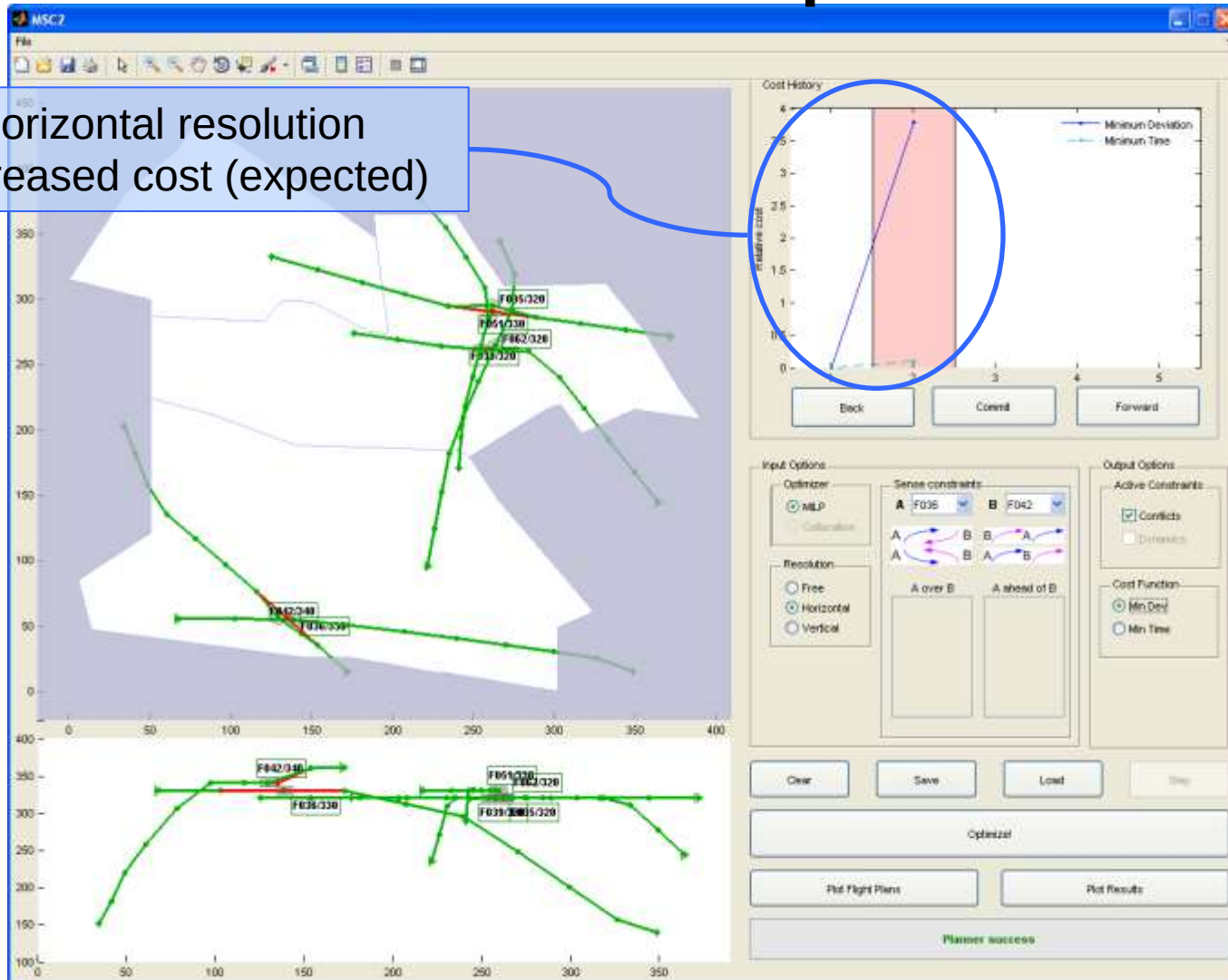
MSC – Step 2

- Request horizontal resolution



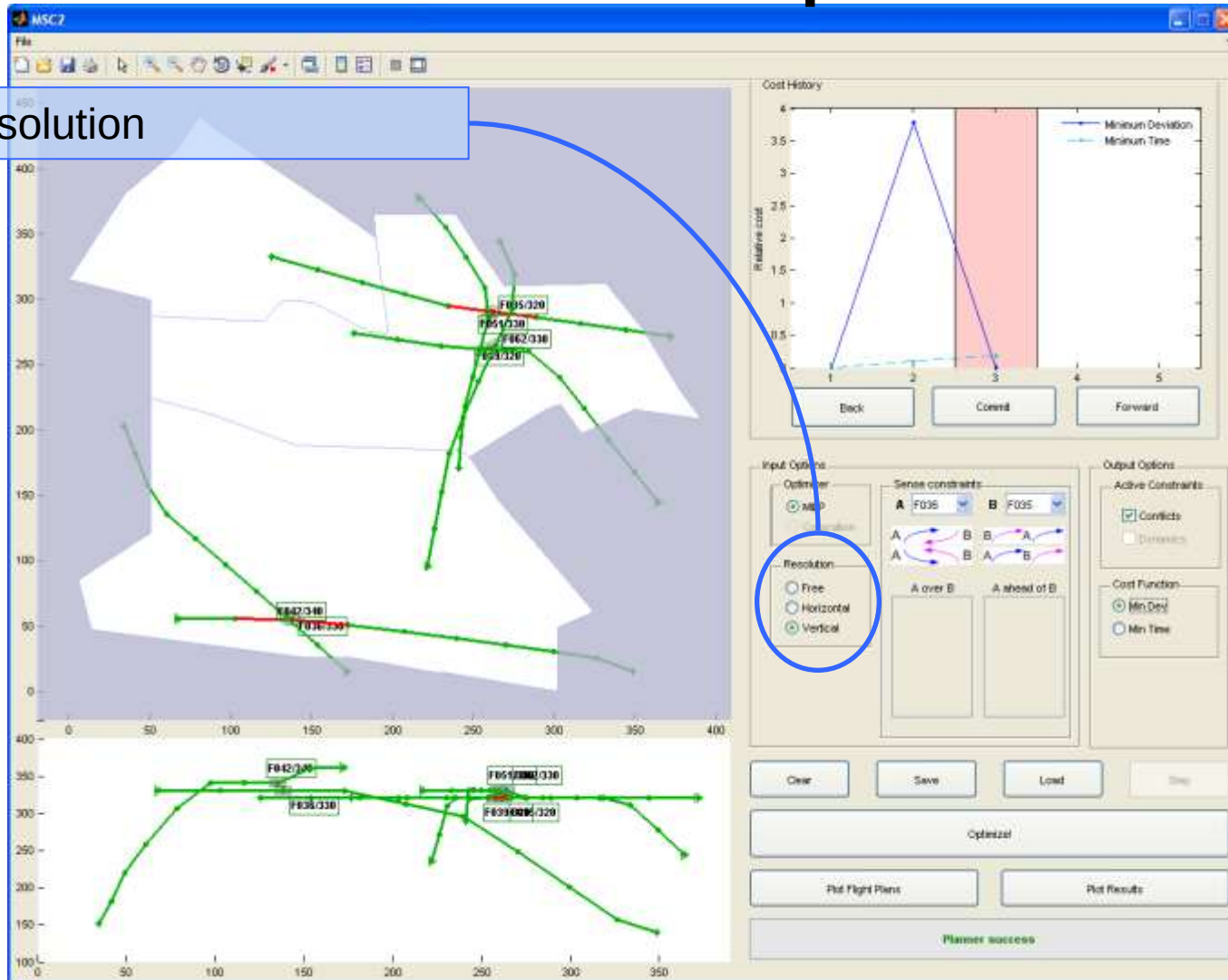
MSC – Step 2

- Request horizontal resolution
 - Increased cost (expected)



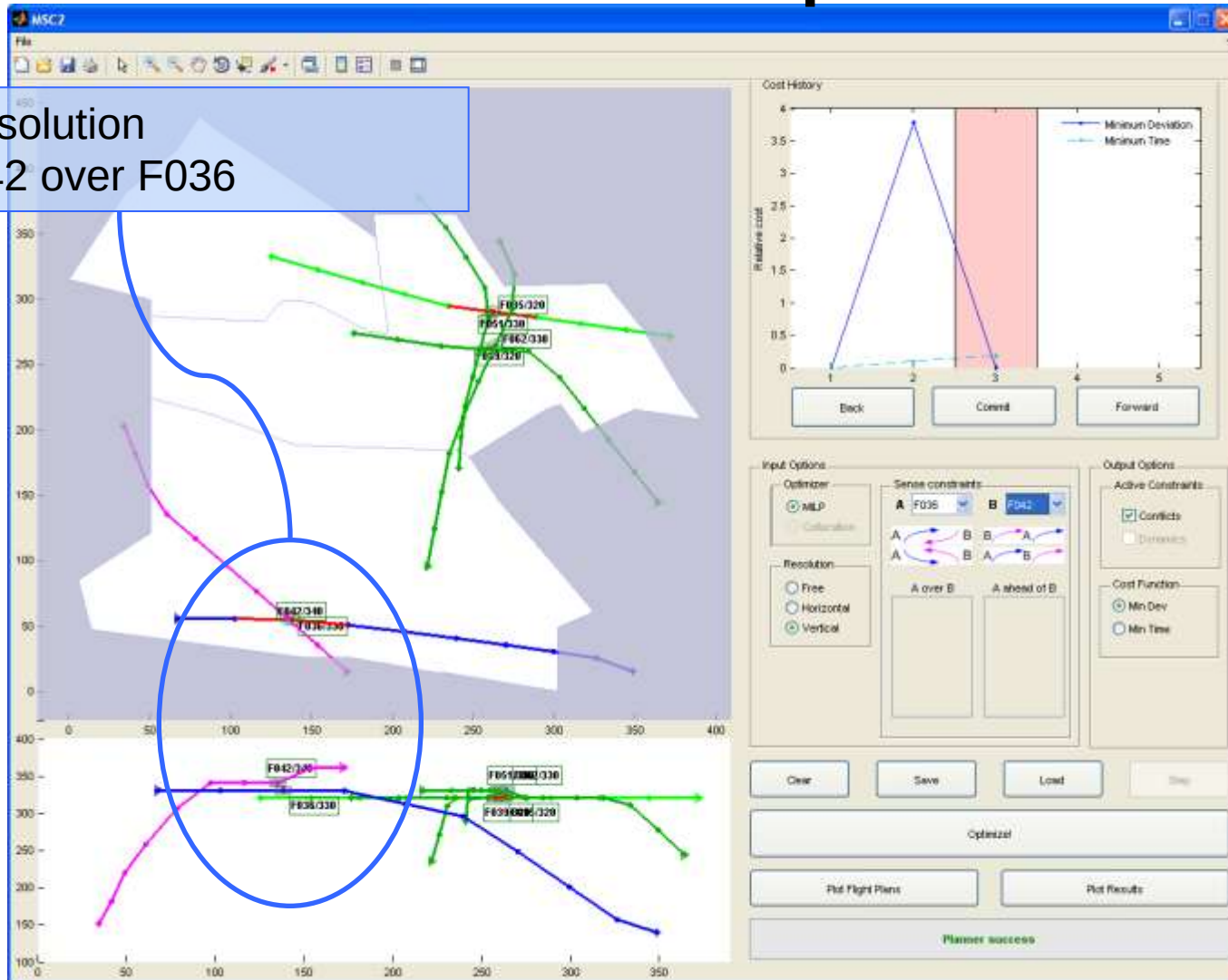
MSC – Step 3

- Vertical resolution



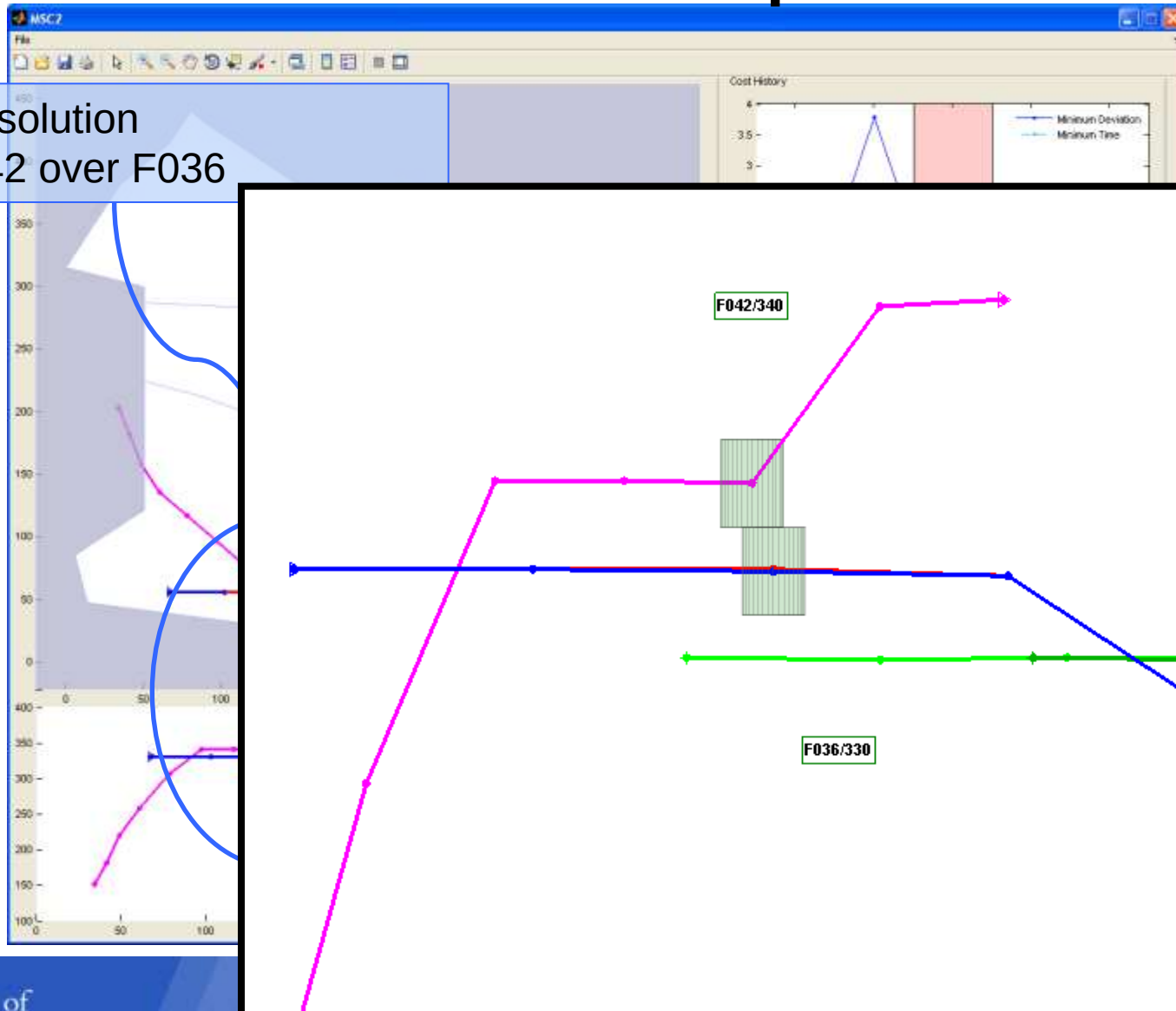
MSC – Step 3

- Vertical resolution
 - F042 over F036



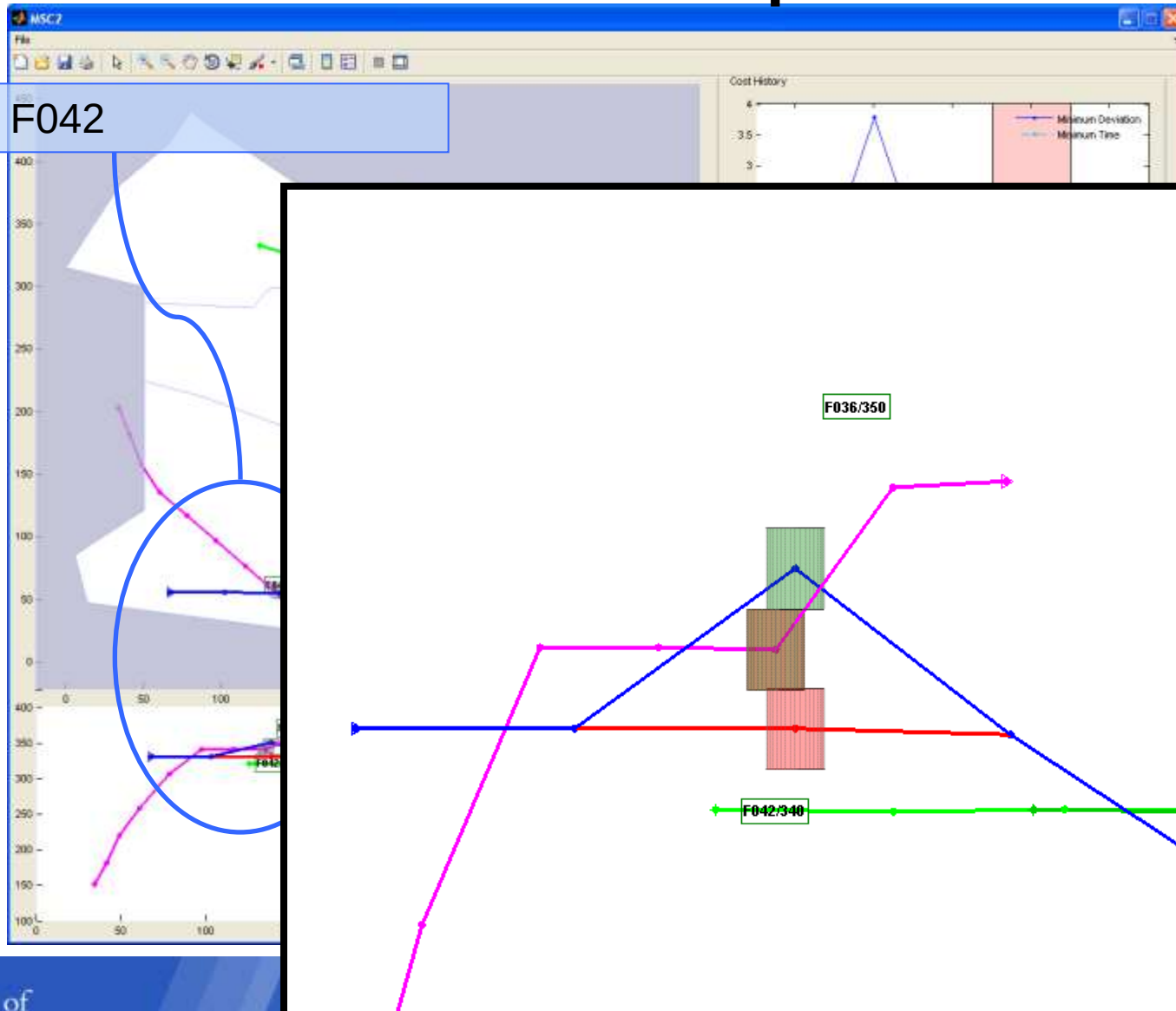
MSC – Step 3

- Vertical resolution
 - F042 over F036



MSC – Step 4

- F036 over F042

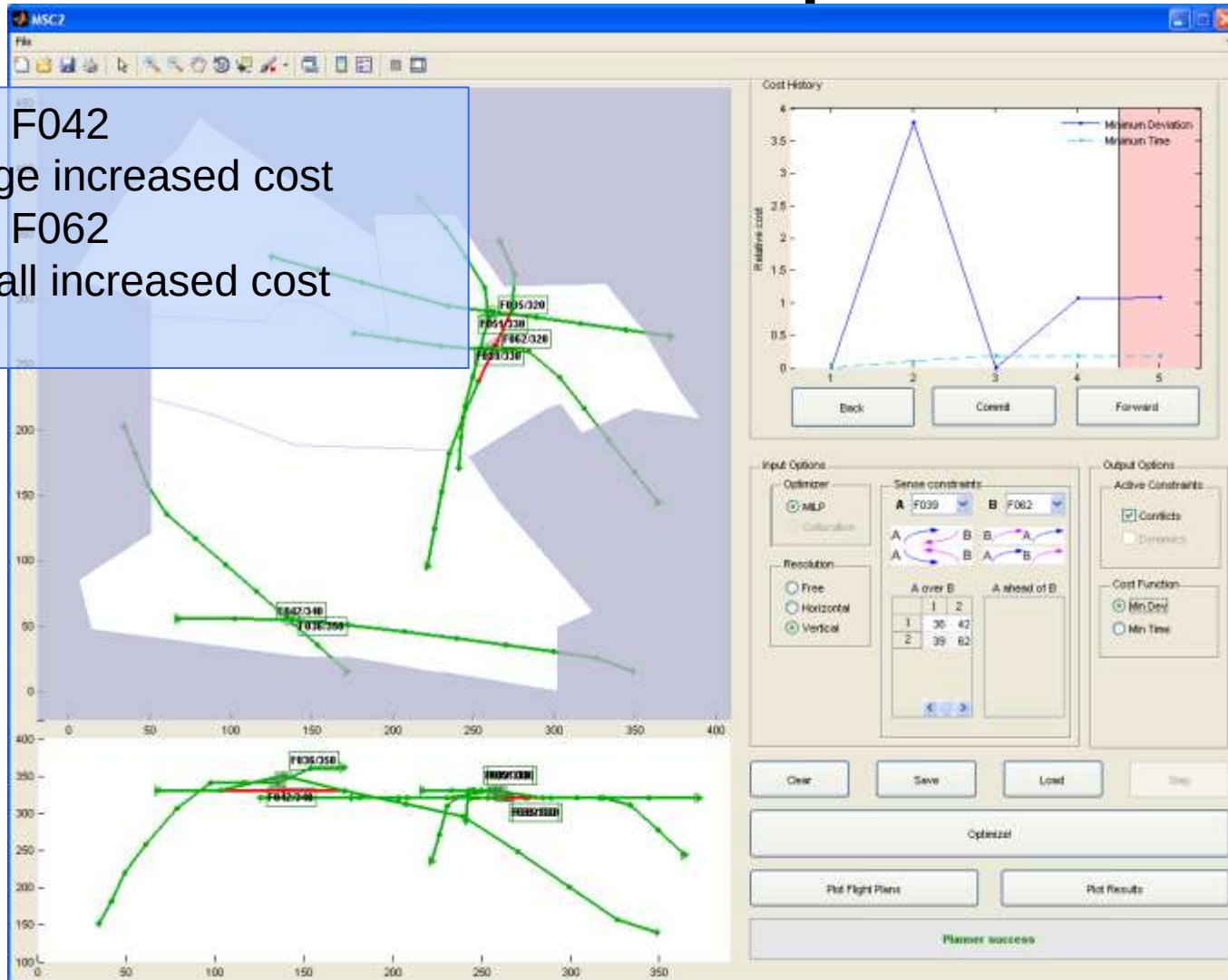


- F036 over F042
 - Large increased cost



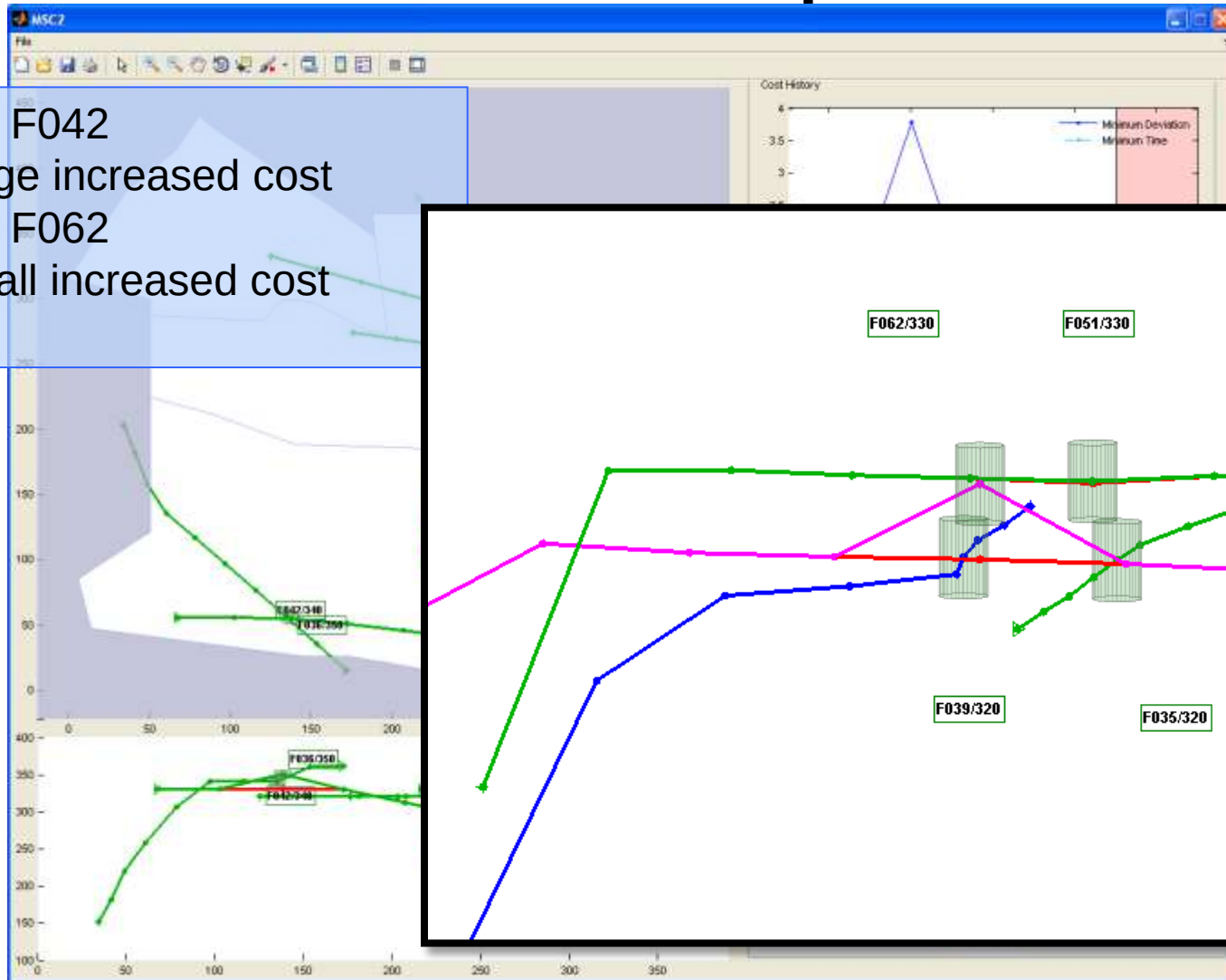
MSC – Step 5

- F036 over F042
 - Large increased cost
- F039 over F062
 - Small increased cost



MSC – Step 5

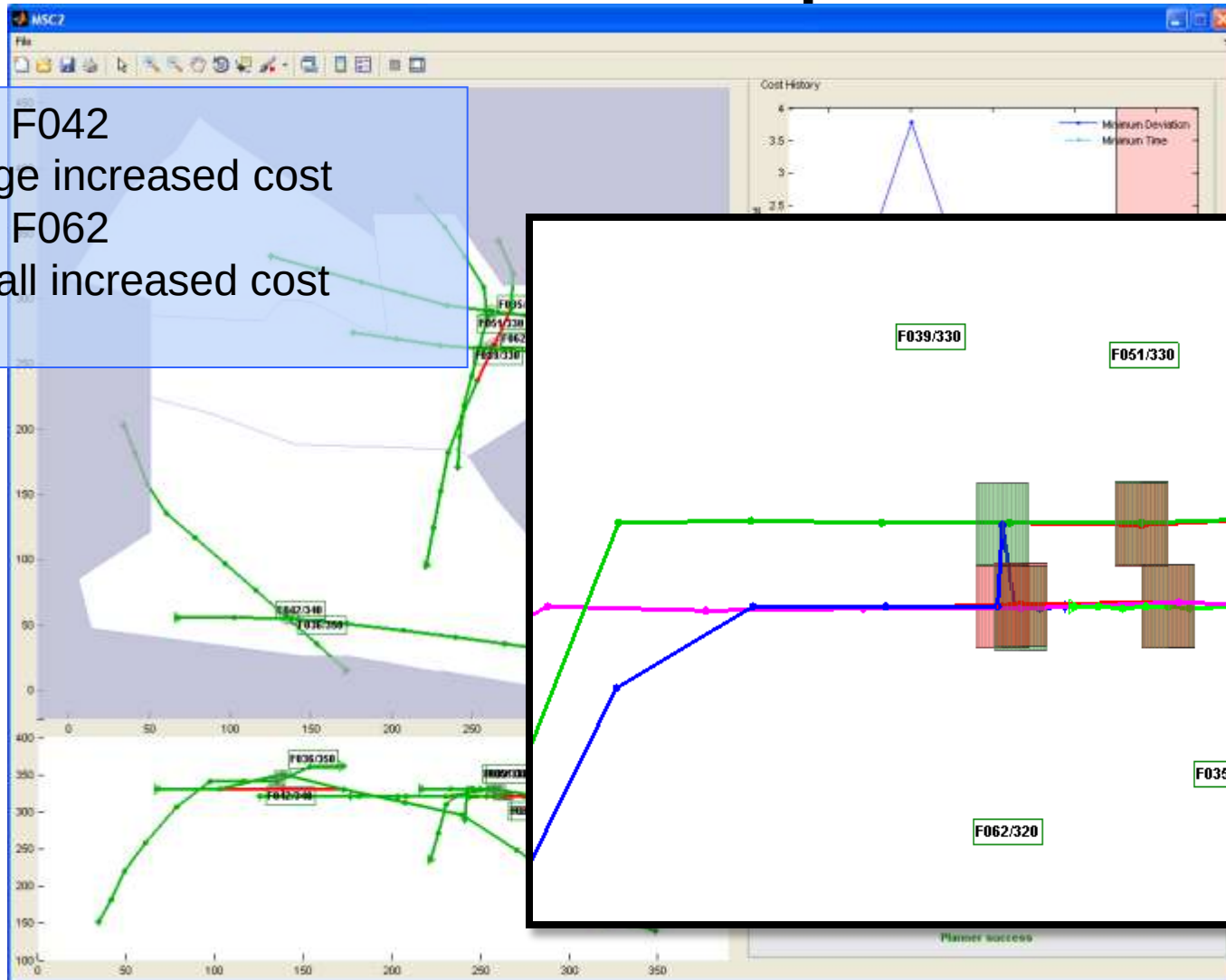
- F036 over F042
 - Large increased cost
- F039 over F062
 - Small increased cost



The screenshot displays the MSC2 software interface. At the top, a menu bar includes 'File', 'Edit', 'View', 'Tools', 'Window', and 'Help'. Below the menu is a toolbar with various icons. The main window is divided into several sections:

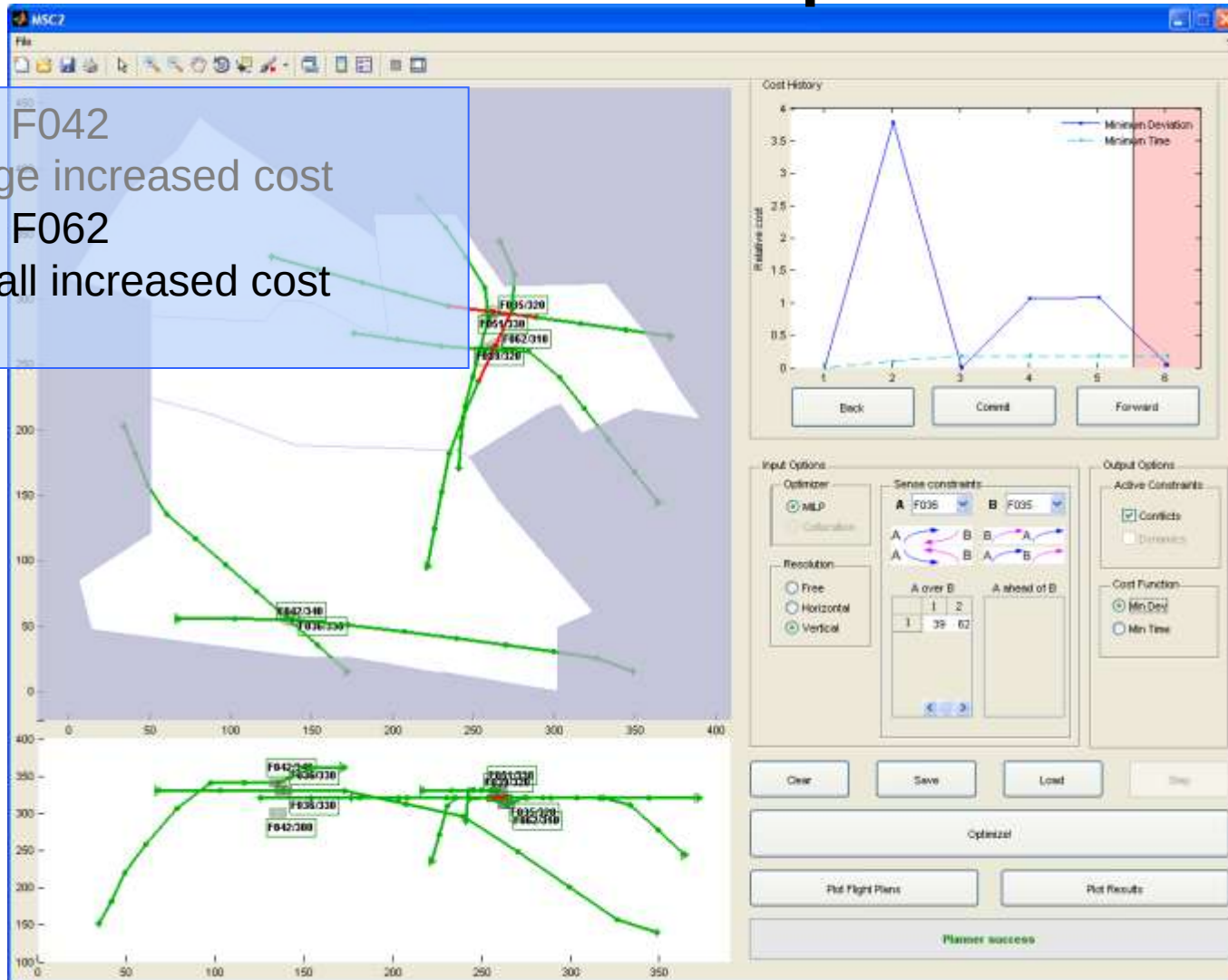
- Top Left:** A blue box contains the text:
 - F042
 - ge increased cost
 - F062
 - all increased cost
- Top Right:** A 'Cost History' graph showing a single peak. The y-axis ranges from 2.5 to 4.0. The x-axis is unlabeled. A legend indicates 'Minimum Deviation' (blue line) and 'Minimum Time' (green line).
- Bottom Left:** A large graph showing multiple green lines representing cost over time. The y-axis ranges from 0 to 400. The x-axis ranges from 0 to 300. Several points are labeled with flight numbers and times, such as F042/318, F036/359, F039/330, F051/330, F062/320, and F036/358.
- Bottom Right:** A detailed graph showing cost over time for various flight segments. The y-axis ranges from 0 to 400. The x-axis ranges from 0 to 350. The graph shows several green lines representing different flight segments. A blue line represents the 'Minimum Deviation' and a green line represents the 'Minimum Time'. The graph is divided into several regions by vertical lines, with labels like F039/330, F051/330, F062/320, and F036/358. A legend at the bottom right indicates 'Planner success'.

-
- The screenshot displays the MSC2 software interface. At the top, a menu bar includes 'File', 'Edit', 'View', 'Tools', 'Window', and 'Help'. Below the menu is a toolbar with various icons. The main window is divided into several sections:
- Top Left:** A blue box contains the text:
 - F042
 - ge increased cost
 - F062
 - all increased cost
 - Top Right:** A 'Cost History' graph showing a single peak. The y-axis ranges from 2.5 to 4.0. The x-axis is unlabeled. A legend indicates 'Minimum Deviation' (blue line) and 'Minimum Time' (green line).
 - Bottom Left:** A large graph showing multiple green lines representing different cost metrics. The y-axis ranges from 0 to 400. The x-axis ranges from 0 to 300. Several data points are labeled with flight numbers and times, such as F042/318, F036/359, F039/330, F051/330, F062/320, and F036/358.
 - Bottom Right:** A detailed graph showing cost breakdowns for various flight segments. The y-axis ranges from 0 to 400. The x-axis ranges from 0 to 350. The graph shows multiple green lines representing different cost metrics. A blue line represents the 'Minimum Deviation' and a green line represents the 'Minimum Time'. The graph is divided into several sections by vertical lines, with labels for flight segments: F039/330, F051/330, F062/320, and F036/358. A legend at the bottom right indicates 'Planner success'.



MSC – Step 6

- F036 over F042
 - Large increased cost
- F039 over F062
 - Small increased cost



MILP Summary

- Well established for trajectory optimization
 - Fast (typically < 1 sec)
 - Robust
 - Globally optimal
- Extended to allow input of user preference for conflict resolution
- Assumptions
 - Linearized dynamics/constraints/cost

Collocation with Polar Sets

- Generalization to a **nonlinear** model is a logical step
 - Collocation method to model aircraft dynamics
 - Polar sets for obstacle avoidance

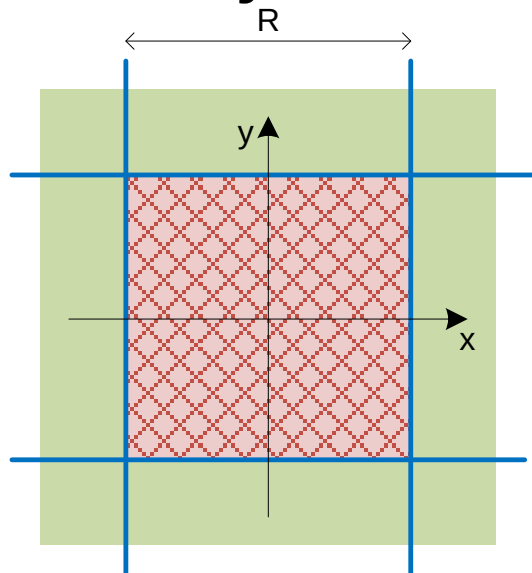


Collocation with Polar Sets

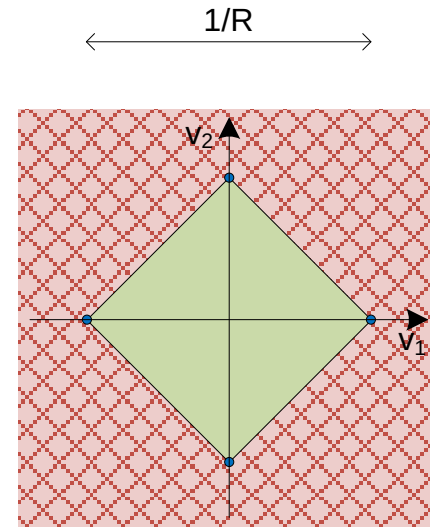
- First we find a point, $\mathbf{y}$, that lies within the polar set of the obstacle: $\mathbf{x} \notin R \iff \mathbf{y}^T \mathbf{x} \geq 1$

$$\mathbf{y} \in R^0$$

- Where $\mathbf{y}$ becomes a decision variable



Cartesian



Polar

Collocation with Polar Sets

- First we find a point, $\mathbf{y}$, that lies within the polar set of the obstacle: $x \notin R \iff \mathbf{y}^T x \geq 1$

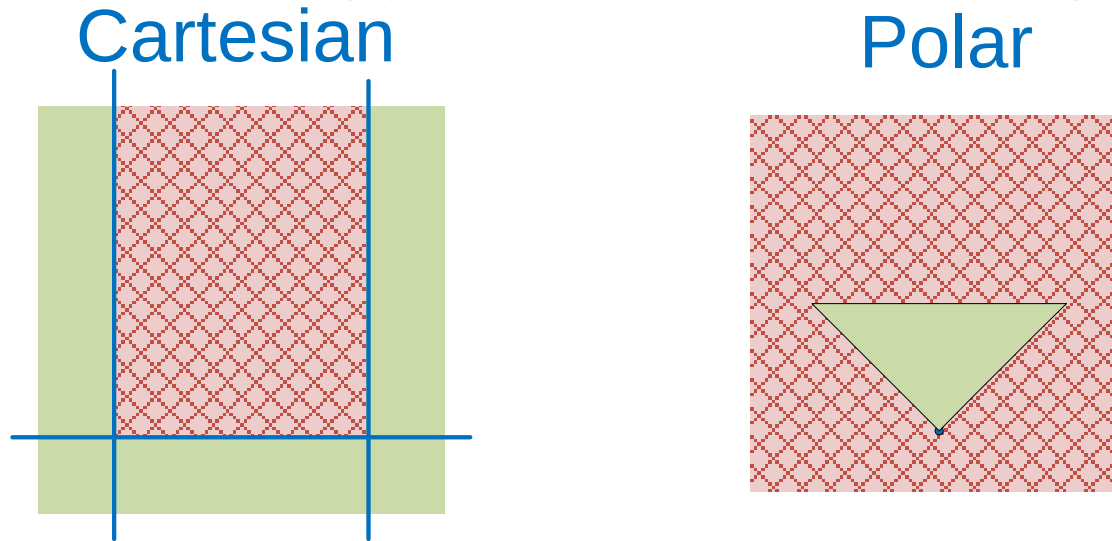
$$\mathbf{y} \in R^0$$

- Next we ensure that the aircraft remains outside of the obstacle (within the polar set) at all time-steps:

$$\mathbf{y}(t)^T \begin{pmatrix} \mathbf{r}(t) - \mathbf{r}_{obs}(t) \\ t - t_{obs} \end{pmatrix} \geq 1 \quad \forall t \in \{2, \dots, N_t\}$$

Polar Set Sense Constraints

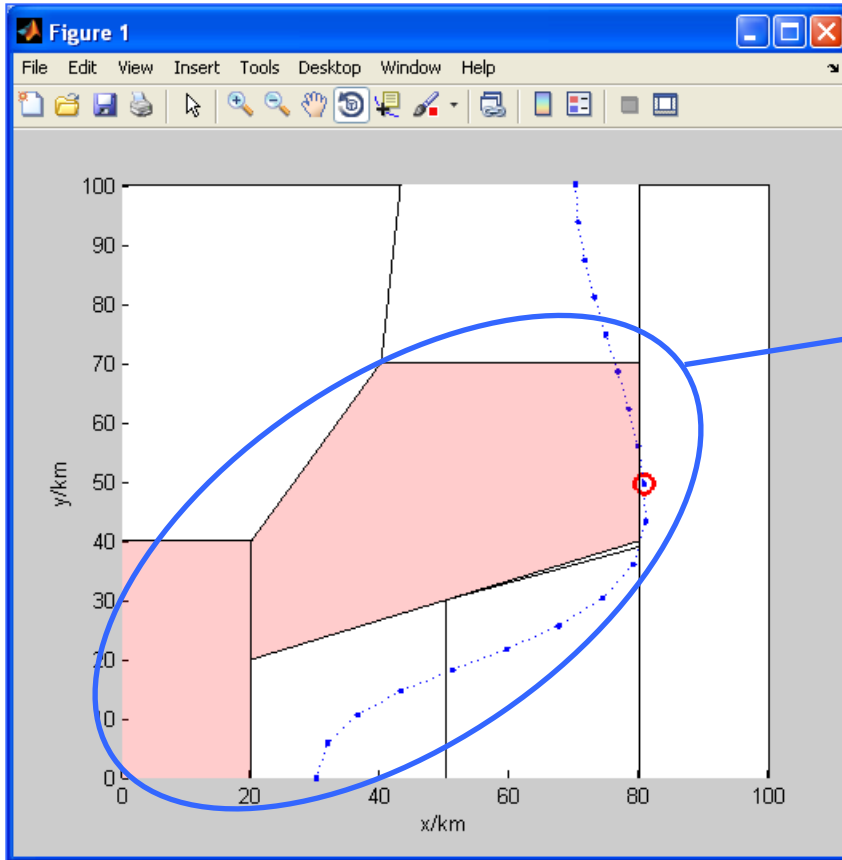
- Equivalent to fixing MILP binaries
- Sense constraints imply removing vertices from the polar set:



- Alternatively we can restrict the location of the point $\mathbf{y}_e(t)$ within the polar set, eg to cause a_1 to pass under an obstacle we would require:

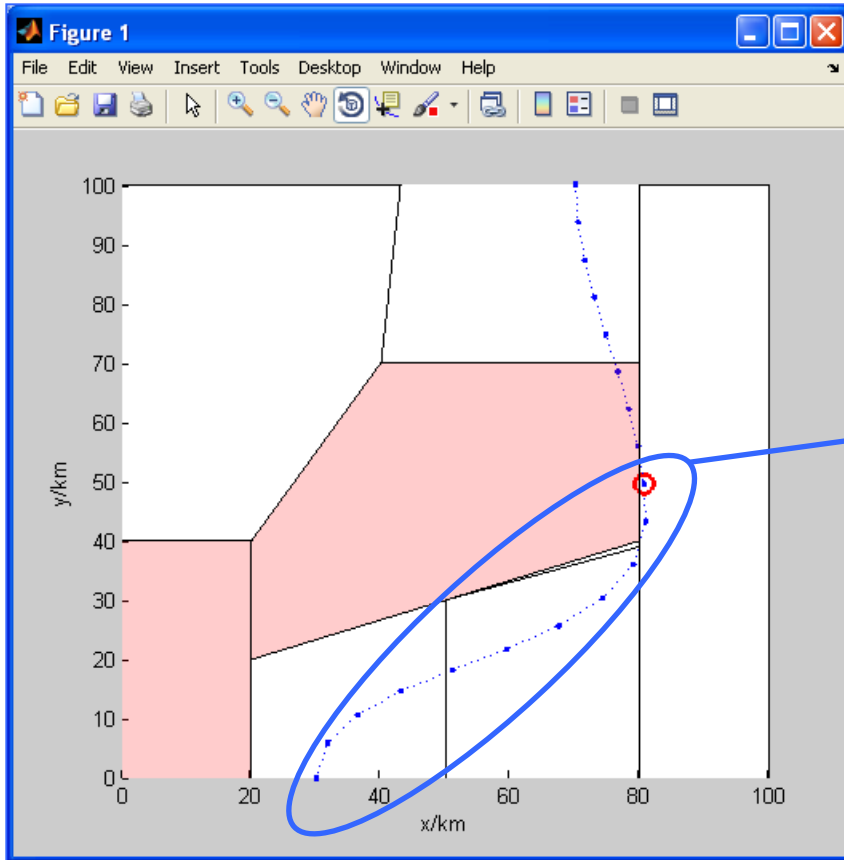
$$y(\tau_1, 3, a_1) \leq 0 \quad \forall \tau_1 \in \{2, \dots, N_t\}$$

4-D Obstacle



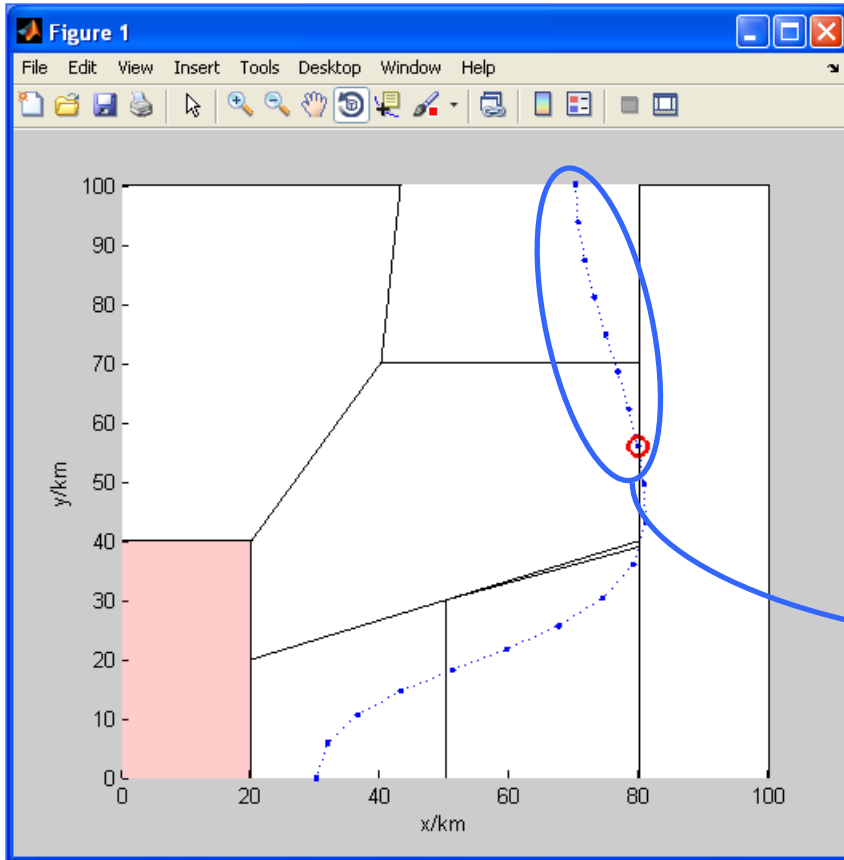
•2 closed sectors

4-D Obstacle



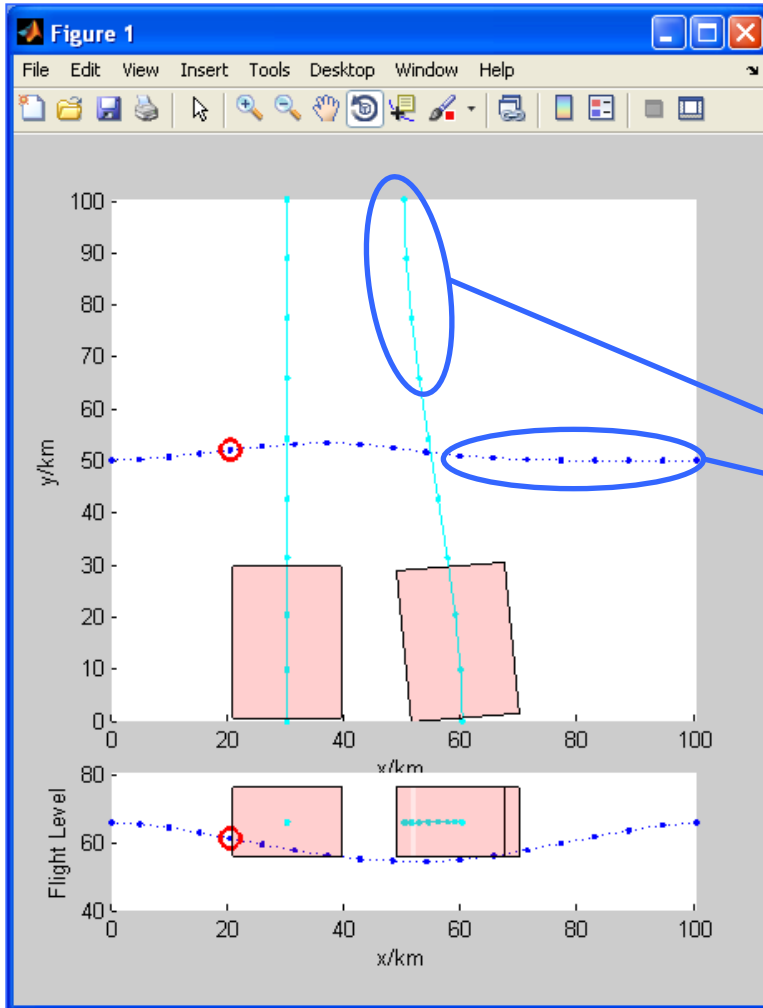
- 2 closed sectors
- Trajectory avoids closed sector

4-D Obstacle



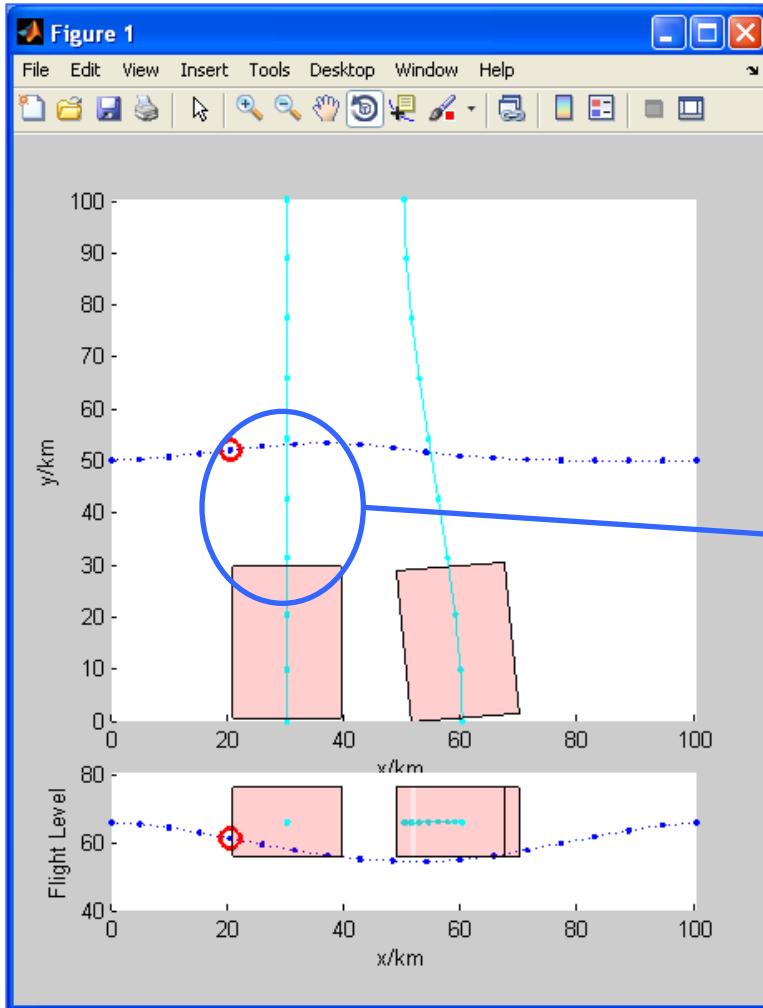
- 2 closed sectors
- Trajectory avoids closed sector
- Sector re-opened
- Trajectory resumes shortest path (through sector)

Collision Avoidance



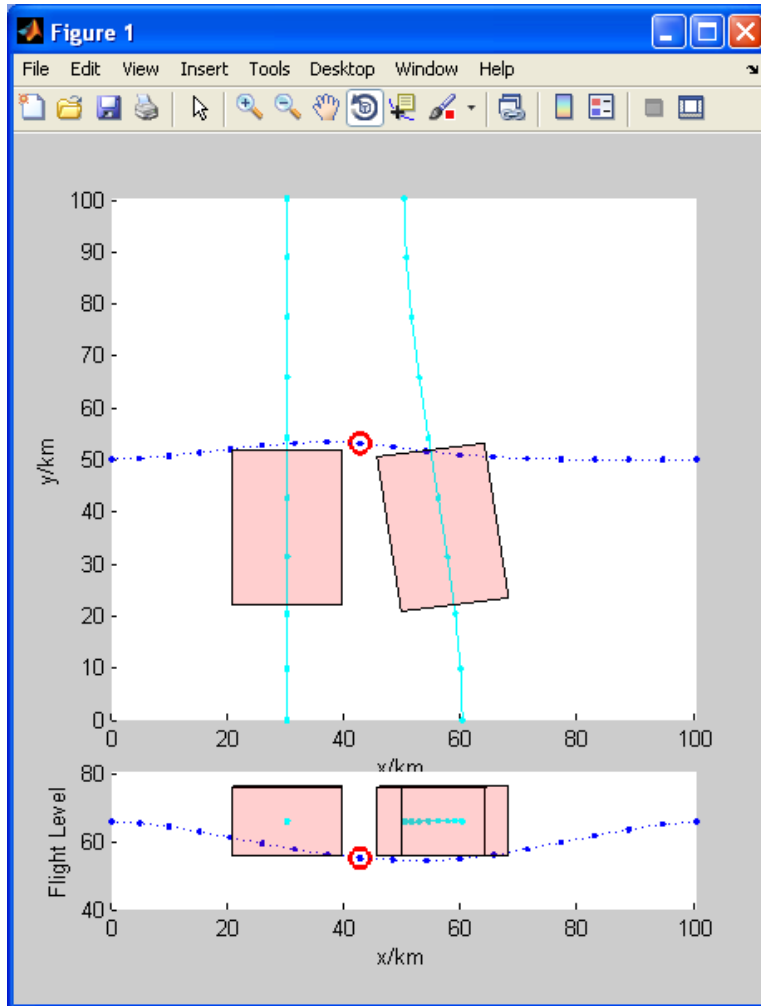
- Planned for F001 (cyan line) – represented by series of temporal obstacles
- Planned for F002 (cyan line)
- Add F003 (blue-dotted line)
- Allows planning over independent time-scales

Collision Avoidance



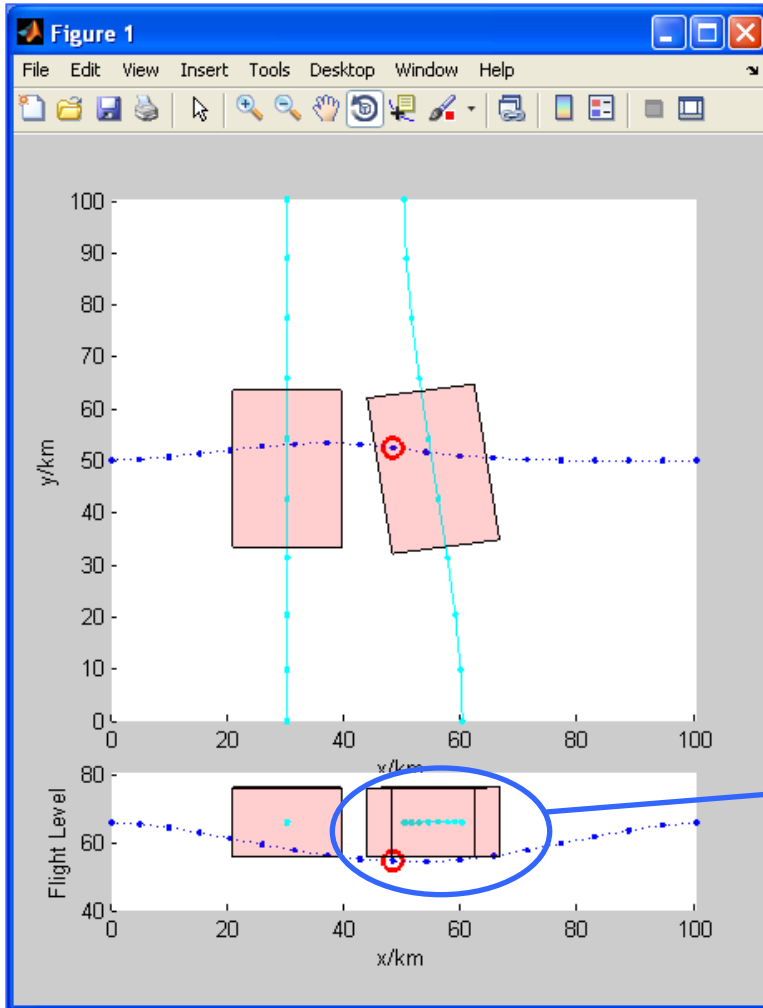
- Planned for first aircraft (cyan line) – represented by series of temporal obstacles
- Planned for F002 (cyan line)
- Add third aircraft (F003)
- Require F003 to pass under F002
- Horizontal separation of F001

Collision Avoidance



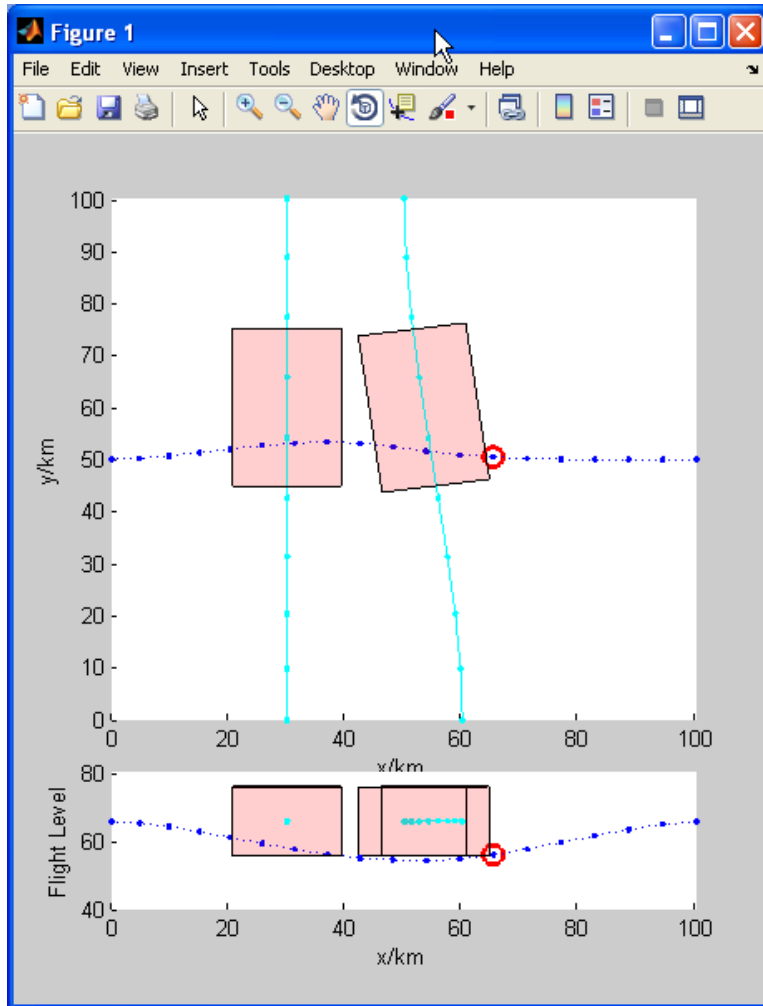
- Planned for first aircraft (cyan line) – represented by series of temporal obstacles
- Planned for F002 (cyan line)
- Add third aircraft (F003)
- Require F003 to pass under F002
- Horizontal separation of F001

Collision Avoidance



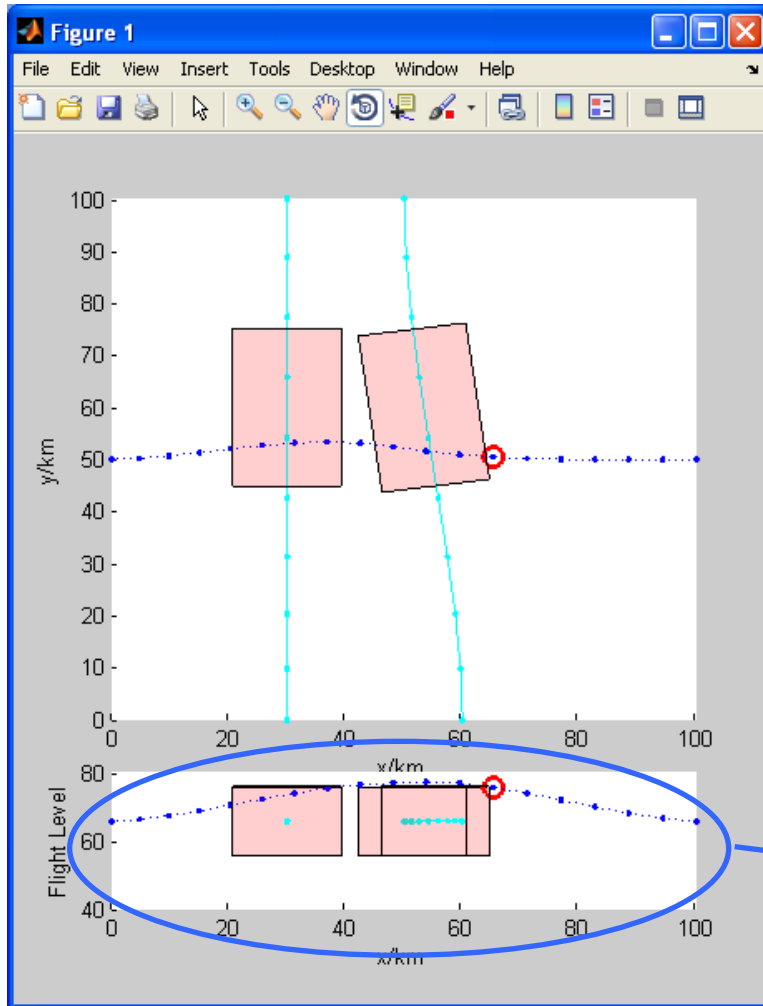
- Planned for first aircraft (cyan line) – represented by series of temporal obstacles
- Planned for F002 (cyan line)
- Add third aircraft (F003)
- Require F003 to pass under F002
- Horizontal separation of F001
- Vertical separation (F003 under F002)

Collision Avoidance



- Planned for first aircraft (cyan line) – represented by series of temporal obstacles
- Planned for F002 (cyan line)
- Add third aircraft (F003)
- Require F003 to pass under other aircraft
- Horizontal separation of F001
- Vertical separation (F003 under F002)

Collision Avoidance



- Planned for first aircraft (cyan line) – represented by series of temporal obstacles
- Planned for F002 (cyan line)
- Add third aircraft (F003)
- Require F003 to pass under other aircraft
- Horizontal separation of F001
- Vertical separation (F003 under F002)
- Alternative sense (F003 over F002)

Conclusion

- Demonstrated **two models** that incorporate sense constraints
- Allows **intuitive human input** in terms of high-level decision making
- While still enabling the optimizer to do what it does best: **design efficient 4D trajectories** subject to avoidance constraints

Thanks!